EXERCISES
IN
ARITHMETIC

new

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EXERCISES IN ARITHMETIC

FOR

ELEMENTARY SCHOOLS.

AFTER THE METHOD OF PESTALOZZI.

UNDER THE SANCTION

OF THE

COMMITTEE OF COUNCIL ON EDUCATION.

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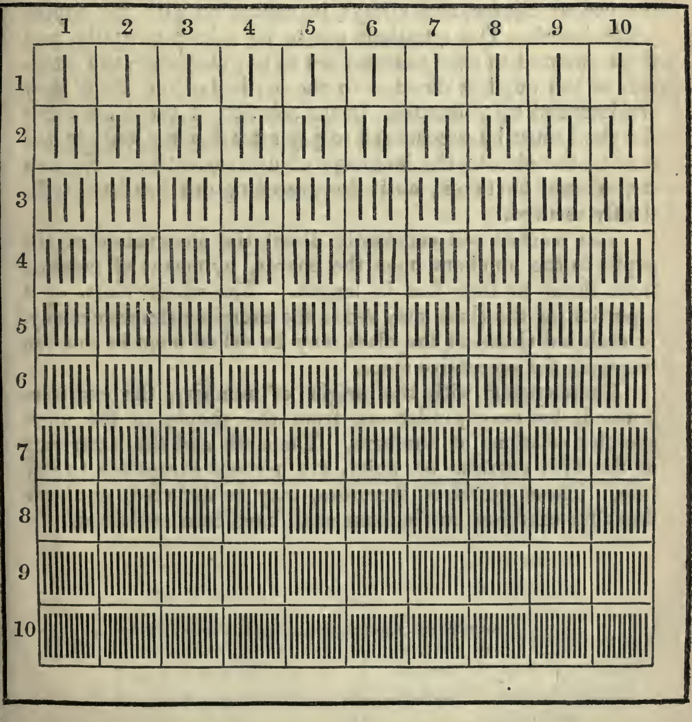
BOOK I.

WHOLE NUMBERS.

INTRODUCTION.

The following exercises are based upon the simple facts, that number consists of distinct units, and that those units may be grouped in different ways.



The Board of Simple Units.

Board of Simple Units. The foregoing figure represents the first board, upon which the relations of simple unity are to be demonstrated. The first line, or line of ones, contains ten marks, or units, placed in separate compartments, or squares. The second line, or line of twos, has two marks, or units, placed in each of the ten squares. The third line, or line of threes, has three marks or units in each square; and so on, concluding with the tenth line, or line of tens, which has ten marks, or units, placed in each square. Thus the whole board is composed of ten lines and ten columns. The number of any line is shown by the numeral at the left hand side of the board, while the number of any column is shown by the numeral at the top of the board.

All the exercises are to be recited by the pupils without the use of figures, and always in connexion with the appropriate board. The questions are to be solved mentally, and those annexed to each exercise are to be given while the attention of the pupil is directed to the particular line which they are intended to illustrate. In the solution of these questions the pupil must be encouraged to pay attention not only to the results, but also to the language of demonstration. No new step should be taken, until the preceding one has been sufficiently secured.

The teacher will constantly direct the observation of the pupils to the numbers upon the boards, by means of pointers from three to four feet in length. The whole class, or a portion of the class, may recite the exercises simultaneously, or each individual in the class may be called upon in rotation to repeat the successive steps.

Commencing with the origin of numbers, the exercises proceed, by easy gradations, from the simple to the more complex relations of numbers. The sixth and the latter part of the fifth exercise on unity, are perhaps exceptions to this rule. These exercises, therefore, may be deferred, until the first four exercises on fractions shall have been explained.

FIRST EXERCISE.

Addition and Subtraction.

The Board of Simple Units being placed in front of the class, the teacher commences this exercise by requiring the pupils to count, in the ordinary way, the marks upon the first line, and afterwards in succession the numbers in each of the different compartments of the other lines.

This simple process must be repeated until the children thoroughly comprehend the construction of the board, and can name the number of marks contained in any square. The class may then proceed with the following tables, first reciting the column of Addition and then repeating it in connection with the column of Subtraction; the teacher being careful always to point out the marks corresponding with the numbers referred to. Thus, whilst the pupils say "4 and 3 make 7," the teacher, with the left-hand rod, counts off four marks, and then, with the other rod, counts off three more. In going over the tables of this exercise for the first time, the marks are to be mentioned in connexion with the number, as for example, "3 marks and 2 marks make 5 marks." In the following Table the first line of squares, or the first and second columns on the Board of Units, is to be referred to.

Addition.	Subtraction.
By Ones.	By Ones.
1 and 1 are 2	1 from 2, and 1 remains
2 and 1 are 3	1 from 3, and 2 remain
3 and 1 are 4	1 from 4, and 3 remain
And so on.	And so on.
By Twos.	By Twos.
1 and 2 are 3	2 from 3, and 1 remains
2 and 2 are 4	2 from 4, and 2 remain
3 and 2 are 5	2 from 5, and 3 remain
And so on.	And so on.
By Threes.	By Threes.
1 and 3 are 4	3 from 4, and 1 remains
2 and 3 are 5	3 from 5, and 2 remain
3 and 3 are 6	3 from 6, and 3 remain
4 and 3 are 7	3 from 7, and 4 remain
And so on.	And so on.

Addition.	Subtraction.
By Fours.	By Fours.
1 and 4 are 5	4 from 5, and 1 remains
2 and 4 are 6	4 from 6, and 2 remain
3 and 4 are 7	4 from 7, and 3 remain
4 and 4 are 8	4 from 8, and 4 remain
And so on.	And so on.

By Fives.	By Fives.
1 and 5 are 6	5 from 6, and 1 remains
2 and 5 are 7	5 from 7, and 2 remain
3 and 5 are 8	5 from 8, and 3 remain
And so on.	And so on.

In like manner the tables may be extended to the addition and subtraction by sixes, sevens, &c. When the pupils have been sufficiently practised in these tables, they may then be repeated in an inverted form; for example, inverting the form of the last line, we shall have,

5 and 1 are 6	1 from 6, and 5 remain
5 and 2 are 7	2 from 7, and 5 remain
5 and 3 are 8	3 from 8, and 5 remain
And so on.	And so on.

These tables are so constructed that every successive addition is obtained from the preceding sum by increasing that sum by unity; so that the pupil remembering, for example, that 3 and 5 make 8, readily concludes that 3 and 6, or 6 and 3, make 9. Also in the repetition of the same numbers, if the pupil know that 3 fives are 15, he may infer that 4 fives are 15 and 5 more, that is, 20.

Second Line.

Addition.	Subtraction.
Of Twos.	Of Twos.
2 and 2 are 4	2 from 4, and 2 remain
4 and 2 are 6	2 from 6, and 4 remain
6 and 2 are 8	2 from 8, and 6 remain
8 and 2 are 10	2 from 10, and 8 remain
And so on.	And so on.

Third Line.

Addition. Of Threes.	Subtraction. Of Threes.
3 and 3 are 6	3 from 6, and 3 remain
6 and 3 are 9	3 from 9, and 6 remain
9 and 3 are 12	3 from 12, and 9 remain
12 and 3 are 15	3 from 15, and 12 remain
And so on.	And so on.

Fourth Line.

Of Fours.	Of Fours.
4 and 4 are 8	4 from 8, and 4 remain
8 and 4 are 12	4 from 12, and 8 remain
12 and 4 are 16	4 from 16, and 12 remain
And so on.	And so on.

The other lines are to be gone over in a similar manner.

The teacher now proceeds to show the connection between multiplication and addition.

Repetition of Ones. First Line.

1 and 1 are twice 1, or 2
1 and 1 and 1 are 3 times 1, or 3
1 and 1 and 1 and 1 are four times 1, or 4
1 and 1 and 1 and 1 and 1 are 5 times 1, or 5
And so on.

Repetition of Twos. Second Line.

2 and 2 are twice 2, or 4
2 and 2 and 2 are 3 times 2, or 6
2 and 2 and 2 and 2 are 4 times 2, or 8
2 and 2 and 2 and 2 and 2 are 5 times 2, or 10
And so on.

Repetition of Threes. Third Line.

3 and 3 are twice 3, or 6
3 and 3 and 3 are 3 times 3, or 9
3 and 3 and 3 and 3 are 4 times 3, or 12
3 and 3 and 3 and 3 and 3 are 5 times 3, or 15
And so on.

Repetition of Fours. Fourth Line.

4 and 4 are twice 4, or 8
4 and 4 and 4, are 3 times 4, or 12
4 and 4 and 4 and 4 are 4 times 4, or 16
4 and 4 and 4 and 4 and 4 are 5 times 4, or 20
And so on.

The same process may be performed upon any of the other lines.

As a further exercise in addition, by adding the units contained in any of the columns, we have,

1 and 2 are 3	3 less by 2 are 1
2 and 3 are 5	5 less by 3 are 2
3 and 4 are 7	7 less by 4 are 3
4 and 5 are 9	9 less by 5 are 4
And so on.	And so on.

Again, let it be required to show the different ways in which any number is made up by the addition of two numbers. For example:

1 and 8 are 9	1 and 10 are 11
2 and 7 are 9	2 and 9 are 11
3 and 6 are 9	3 and 8 are 11
4 and 5 are 9	4 and 7 are 11
And so on.	5 and 6 are 11
	And so on.

In like manner we have for subtraction:

10 less by 1 are 9
11 less by 2 are 9
12 less by 3 are 9
13 less by 4 are 9
And so on.

Here, it will be observed, that in order that the sum of two numbers may remain the same, we must increase the one number by the same quantity that we diminish the other, and in like manner, in order that the difference may remain the same, we must increase or diminish the two numbers by the same quantity.

Questions on the First Exercise.

The following questions will show to the teacher some of the various ways in which questions may be put. In order that the results of the foregoing exercise may be thoroughly impressed upon the memory of the pupil, the teacher will find it necessary to extend these questions.

The words, First Line, Second Line, &c., intimate that the teacher must point out on what line of squares on the board the pupils are to seek for an answer to the question proposed.

First Line.

i. How many times must one be repeated to make ten ?.... *Ans.* Ten times.

ii. One and one and one make what number ?....*Ans.* Three.

iii. One from three leaves how many ?....*Ans.* Two.

Second Line.

i. Three twos make what number ?....*Ans.* Six, because two and two make four, and four and two make six.

ii. Two and six make what number ?....*Ans.* Eight, because six and one make seven, and seven and one make eight.

iii. A man receives 2 shillings a day as his wages ; how many shillings will he receive for 6 days' work ?....*Ans.* Twelve.

iv. If John has 2 marbles in one pocket, and 3 in the other ; how many has he altogether ?....*Ans.* Five.

v. Give the addition of twos as far as twenty.

Third Line.

i. Count the squares of glass in the window by threes.

ii. Show in how many ways 5 marbles may be made up.

iii. Give the addition of threes as far as twenty-one.

iv. If 1 article cost three-pence, how much will 5 articles cost ?....*Ans.* Fifteen pence.

Fourth Line.

i. Give the addition of fours as far as twenty.

ii. Repeat 4 six times.

iii. If I have 4 shillings in my pocket, and afterwards pay 3 shillings away ; how many shillings have I left ?....*Ans.* One.

Fifth Line.

i. John had 5 nuts, and gave 2 of them away; how many had he left?....*Ans.* Three.

ii. What must I take from five-pence to leave three-pence?....*Ans.* Two-pence.

iii. Count the marks on the fifth line by fives.

SECOND EXERCISE.

Multiplication and Division.

Addition is an operation by which we collect two, or more numbers into one whole. Subtraction is the reverse of Addition. The sign of Addition is +, or plus: that of Subtraction —, or minus.

When *equal* numbers are to be added, we proceed by a more compendious method, called Multiplication, as for instance 4 and 4 and 4, or the number 4 repeated 3 times, equals 12. This operation is called the multiplication of 4 by 3, and it is symbolically expressed by $3 \times 4 = 12$. The numbers 4 and 3 are called the *factors* of 12; the number 4 receives the name of the *multiplicand*, 3 that of the *multiplier*, and 12 that of the *product*.

To multiply a number therefore we have to add it as many times as there are units in the multiplier.

The first column of the following table shews the manner in which "the Multiplication Table" is formed, and is a recapitulation of some of the results obtained in the first exercise. Thus on the second line, each successive product is obtained by adding 2 to the preceding product; and on the third line, each successive product is obtained by adding 3 to the preceding one, and so on. In the second column the operation is reversed, that is, the units contained in the product are resolved into factors, and then one of these factors is said to be contained in the product a certain number of times. In the third column, the pupil is led to consider the relation of one of these factors to the whole product. By these progressive steps, a distinct idea is conveyed to the pupil of the meaning of the terms, one-half, one-third, one-fourth, &c., considered in relation to units.

When this exercise is introduced to the class, the first column is to be gone over by itself; and then repeated in con-

nexion with the second column; and lastly, the three columns, or the first and third only, may be taken together.

The results contained in this exercise are to be given by the children on observing the relations pointed out by the teacher on the Board of Simple Units. As an illustration, let us take the fifth line: When the teacher points to the first square, the children say "once 5 is 5; 5 are contained in 5 once;" then in pointing to the second square, the children say, "twice 5 are 10; 5 are contained in 10 twice; the half of 10 is 5," and so on. If the children do not immediately give these results, the teacher must guide them by such suggestive questions as the following:

How many fives are here?....*Ans.* Two fives.

What do 2 fives make?....*Ans.* Ten.

How many fives make up 10?....*Ans.* Two fives.

How many times is 5 contained in 10?....*Ans.* Twice.

If ten-pence be equally divided between two boys, what part of the ten-pence does each receive?....*Ans.* One-half.

What is meant by the fifth of 10?....*Pupil.* That if 10 be separated into five equal parts, then the fifth of 10 is the number of units contained in one of these parts: thus on the second line, I see that 10 is composed of five 2's, therefore 2 is the fifth of 10.

What then is the fifth of any number?....*Pupil.* The number of units contained in one of the five equal parts into which the given number is divided.

First Line.

First Column.	*Second Column.*	*Third Column.*
Twice 1 are 2	1 is contained in 2 twice	The half of 2 is 1
3 times 1 are 3	1 is contained in 3 three times	The third of 3 is 1
4 times 1 are 4	1 is contained in 4 four times	The fourth of 4 is 1
5 times 1 are 5	1 is contained in 5 five times	The fifth of 5 is 1
And so on to	And so on to	And so on to
10 times 1 are 10	1 is contained in 10 ten times	The tenth of 10 is 1

Second Line.

Once 2 is 2	2 are contained in 2 once	
Twice 2 are 4	2 are contained in 4 twice	The half of 4 is 2
3 times 2 are 6	2 are contained in 6 three times	The third of 6 is 2
4 times 2 are 8	2 are contained in 8 four times	The fourth of 8 is 2
5 times 2 are 10	2 are contained in 10 five times	The fifth of 10 is 2
And so on to	And so on to	And so on to
10 times 2 are 20	2 are contained in 20 ten times	The tenth of 20 is 2

Third Line.

First Column.	*Second Column.*	*Third Column.*
Once 3 is 3	3 are contained in 3 once	
Twice 3 are 6	3 are contained in 6 twice	The half of 6 is 3
3 times 3 are 9	3 are contained in 9 three times	The third of 9 is 3
4 times 3 are 12	3 are contained in 12 four times	The fourth of 12 is 3
5 times 3 are 15	3 are contained in 15 five times	The fifth of 15 is 3
And so on to	And so on to	And so on to
10 times 3 are 30	3 are contained in 30 ten times	The tenth of 30 is 3

And so on through the fourth, fifth, sixth, seventh, and eighth lines.

Ninth Line.

Once 9 is 9	9 are contained in 9 once	
Twice 9 are 18	9 are contained in 18 twice	The half of 18 is 9
3 times 9 are 27	9 are contained in 27 three times	The third of 27 is 9
4 times 9 are 36	9 are contained in 36 four times	The fourth of 36 is 9
5 times 9 are 45	9 are contained in 45 five times	The fifth of 45 is 9
And so on to	And so on to	And so on to
10 times 9 are 90	9 are contained in 90 ten times	The 10th of 90 is 9

Tenth Line.

Once 10 is 10	10 are contained in 10 once	
Twice 10 are 20	10 are contained in 20 twice	The half of 20 is 10
3 times 10 are 30	10 are contained in 30 three times	The third of 30 is 10
4 times 10 are 40	10 are contained in 40 four times	The fourth of 40 is 10
5 times 10 are 50	10 are contained in 50 five times	The fifth of 50 is 10
And so on to	And so on to	And so on to
10 times 10 are 100	10 are contained in 100 ten times	The tenth of 100 is 10

In this table Division is treated as the reverse operation of Multiplication; but since the latter is an extension of the rule of Addition, so, in like manner, Division may be regarded as an extension of the rule of Subtraction. Thus to divide 9 by 3, is the same thing as finding the number of times that 3 may be subtracted from 9; first, we say 3 from 9 and 6 remain, then 3 from 6 and 3 remain, and lastly, 3 from 3 and nothing remains; hence we ascertain that 3 is contained in 9 three times without any remainder. Again, in the division of 7 by 2, we find, that after 3 twos have been taken from 7, there is 1 remaining, that is, 2 is contained in 7 three times, and 1 for the remainder.

It will be useful at this stage of the pupil's progress to

show that quantities may be multiplied in any order*. Thus, while the teacher points with one rod to the eighth square of the sixth line, and with another rod to the sixth square of the eighth line, the pupil sees that

48 are 8 times 6, or 6 times 8.

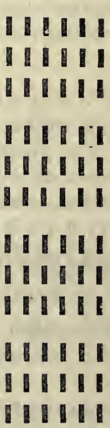
Again, on the fourth and sixth lines,
24 are 6 times 4, or 4 times 6.

And so on to any other numbers.

It will also be instructive to perform the addition and subtraction of different combinations of numbers†; for example, on the second, third, and fourth lines.

3 twos and 2 twos are 5 twos	5 twos less by 2 twos leave 3 twos
4 threes and 3 threes are 7 threes	7 threes less by 3 threes leave 4 threes
5 fours and 2 fours are 7 fours	7 fours less by 2 fours leave 5 fours
7 fives and 3 fives are 10 fives	10 fives less by 3 fives leave 7 fives
4 sixes and 2 sixes are 6 sixes	6 sixes less by 2 sixes leave 4 sixes
8 nines and 3 nines are 11 nines	11 nines less by 3 nines leave 8 nines
&c.	&c.

* This important principle may be further demonstrated by the following method. Let it be required to shew that the product of the numbers 3, 4, and 6, will be the same in whatever order they may be multiplied, that is, that $3\times4\times6=4\times3\times6=3\times6\times4=$ &c. The product of these numbers will be shewn by the marks in the accompanying figure, where, in each group, 6 is taken 3 times, or 3 is taken 6 times; and the number in each group is repeated four times, making up altogether the number 72. Taking, therefore, the lines horizontally, we have 6 taken 3×4 times or 12 times; taking them vertically, we have 3 taken 4×6 times, or 3 taken 24 times; and so on to every possible combination.



† In this manner it is shewn, that any quantity taken a certain number of times, will be the same as the parts of that quantity taken the same number of times; for example $5\times(6+3)=5\times6+(5\times3)$. Again 5 is made up of 3 and 2; 7 times 5 therefore will be the same as 7 times 3, added to 7 times 2, or $7\times(3+2)=7\times3+7\times2$.

As a further and more complete illustration of this useful principle, we have—

On the ninth, eighth, and first lines.
9 are 8 and 1.

Twice 9 are twice 8 and twice 1	Twice 8 and twice 1 are twice 9
3 times 9 are 3 times 8 and 3 times 1	3 times 8 and 3 times 1 are 3 times 9
4 times 9 are 4 times 8 and 4 times 1	4 times 8 and 4 times 1 are 4 times 9

And so on to 5, 6, 7...... times 9.

On the ninth, seventh, and second lines.
9 are 7 and 2.

Twice 9 are twice 7 and twice 2	Twice 7 and twice 2 are twice 9
3 times 9 are 3 times 7 and 3 times 2	3 times 7 and 3 times 2 are 3 times 9

And so on to 4, 5, 6...... times 9.

On the ninth, sixth, and third lines.
9 are 6 and 3.

Twice 9 are twice 6 and twice 3	Twice 6 and twice 3 are twice 9
3 times 9 are 3 times 6 and 3 times 3	3 times 6 and 3 times 3 are 3 times 9

And so on to 4, 5, 6...... times 9.

On the ninth, fifth, and fourth lines.
9 are 5 and 4.

Twice 9 are twice 5 and twice 4	Twice 5 and twice 4 are twice 9
3 times 9 are 3 times 5 and 3 times 4	3 times 5 and 3 times 4 are 3 times 9

And so on to 4, 5, 6...... times 9.

We may also operate upon 2, 3, 4, 5, &c. in the same manner as we have here done upon 9.

The principle here exhibited, so important in the demonstration of the rule of multiplication according to the decimal notation, is beautifully shewn by the unity board. Thus, for "3 times 9 are 3 times 5 and 3 times 4," &c. we have

On the fifth line	IIIII	IIIII	IIIII	that is, 3 times 5
On the fourth	IIII	IIII	IIII	that is, 3 times 4
On the ninth	IIIIIIIII	IIIIIIIII	IIIIIIIII	,, 3 times 9

Many of the answers given to the following questions may not be such as the pupils would give in the first instance, but are intended to show how the teacher should lead them to apply the method of reasoning employed in the exercise to the solution of any particular question that may be proposed. The same observation will be applicable to the questions appended o all the subsequent exercises.

Questions on the Second Exercise.

First Line.

i. How many units are contained in 5 ?....*Ans.* Five.

ii. What is the fifth of 5 ?....*Ans.* One. Because 1 taken five times produces 5.

Second Line.

i. Where do you show the product of five times 2 ?.... *Ans.* The units contained in the first five squares of the second line show that five times 2 are 10.

ii. What is the fourth of 8 ?....*Ans.* Two. Because if 8 be separated into four equal parts, we shall have two units in each part.

iii. What operation must be performed upon the number 2 so as to produce 12 ?....*Ans.* Two must be added to itself 6 times, that is, multiplied by 6, to produce 12.

iv. What is meant by dividing 14 by 2 ?....*Ans.* It is finding the number of times that 2 must be repeated to make up 14. Thus 7 times 2 are 14, therefore 2 is contained in 14 seven times.

v. Distribute 8 into as many twos as possible.....*Ans.* 8 = 2 and 2 and 2 and 2, that is, 8 is made up of 4 twos.

vi. How many twos can you take out of 6 ?

Third Line.

i. Knowing that 4 times 3 make 12, how do you find the product of 5 times 3 ?....*Ans.* By adding 3 to 12, which gives 15 for 5 times 3.

ii. Where are 12 units distributed into four equal collections of units ?....*Ans.* In the first four compartments of the third line.

iii. What is the sixth part of 18 ?....*Ans.* 3: because 6 times 3 are 18; and therefore 3 is the sixth of 18.

iv. Find a common divisor of 9 and 12.....*Ans.* 3: because 9 and 12 are both formed upon the line of threes.

v. What is the common divisor of 15 and 21 ?....*Ans.* 3: because 15 are 5 threes, and 21 are 7 threes.

vi. Of what numbers is 12 the multiple?....*Ans.* 2, 3, 4, and 6.

vii. 9 times 3 are 4 times 3 and how many threes more?

Similar questions may be given on any of the other lines.

From time to time the teacher will introduce terms commonly used in arithmetic, taking care always to explain their meaning. Thus in the fourth and sixth questions the terms "common divisor" and "multiple" are introduced; and it may be there explained that a common divisor of two or more numbers is a smaller number, which is contained in each of them an exact number of times. In like manner it may be stated that one number is a "multiple" of another when the former contains the latter two or more times without any remainder; thus, 9, 12, and 15, are multiples of 3; while 3 is a common divisor of 9, 12, and 15.

The teacher may also gradually introduce questions relating to money, weights, and measures, such as the following, giving his pupils sufficient instruction in the common tables on those subjects to enable them to give the proper solutions. A complete knowledge of such tables would not be requisite early in the course of instruction.

Questions on Money, Weights, and Measures.

Second Line.

i. What is the cost of 2 oz. of coffee at 2*d.* per oz.?.... *Ans.* 4*d.*: because twice 2 = 4.

ii. Divide 6*d.* equally amongst 3 boys: how much will each receive?....*Ans.* 2*d.* each: because 2*d.* repeated 3 times produces 6*d.*

iii. If 4 lbs. of coffee cost 8*s.*, what is the price of 1 lb.? ....*Ans.* 2*s.*: because the cost of 1 lb. will be the fourth of the cost of 4 lbs., and the fourth of 8*s.* is 2*s.*

Third Line.

i. What is the cost of 4 oz. of tobacco at 3*d.* per oz.?.... *Ans.* 12*d.*: because 4 times 3 are 12.

ii. What must I pay for half a pound of sugar at 6*d.* per lb.?....*Ans.* 3*d.*: because 6*d.* divided into two equal parts gives 3*d.* for each.

iii. How many pints of ale can I get for 15*d.*, when one pint costs 3*d.*?....*Ans.* 5 pints: because 3 is contained in 15 five times.

Fourth Line.

i. Divide 20*d.* equally amongst 5 persons: how much will each receive?....*Ans.* 4*d.*

ii. If 3 lbs. of tea cost 12*s.*, what is the price of 1 lb.?.... *Ans.* 4*s.*: because 1 lb. will cost the third of 12*s.* which is 4*s.*

iii. What is the fifth of 1*l.*, or 20*s.*?....*Ans.* 4*s.*: because 20 separated into 5 equal parts gives 4 units in each part.

THIRD EXERCISE.

Division when there is a Remainder.

In the foregoing exercise the nature of Division is rendered evident by considering it as the reverse of Multiplication. In the present exercise the pupil proceeds at once to decompose numbers into certain groups of units with remainders; then in the second column he finds the composition of these groups with the remainders; and lastly, in the third column he finds the number of times, with a certain remainder, which these groups of units are contained in the original number of units.

In introducing this exercise, the first column is to be gone over by itself; then the second and third; and, lastly, all three columns are to be recited together.

As an example of the mode of teaching this exercise, we will suppose the teacher to be about to show that "twice 3 and 2 are 8." With the right hand rod, he points off eight units on the third line; then with the left hand rod he shews that these units consist of 2 threes and 2 over, or in other words that "twice 3 and 2 are 8," and conversely, "3 is contained in 8 twice with 2 remaining."

In the course of the exercise the teacher may propose such familiar questions as are likely to lead the pupils to reason on the results obtained. For example:

How many twos are here? (pointing off the seven marks on the second line.)....*Ans.* Three twos.

And what more?....*Ans.* And one more.

Then three twos and one make what number?....*Ans.* Three twos and one make 7.

Then how many twos are contained in, or can be taken out of, 7?....*Ans.* 2 are contained in 7 three times and 1 remaining.

Second Line.

First Column.	*Second Column.*	*Third Column.*
2 are once 2	once 2 are 2	2 are contained in 2, once
3 are once 2 and 1	once 2 and 1 are 3	2 are contained in 3, once and 1 remaining
4 are twice 2	twice 2 are 4	2 are contained in 4, twice
5 are twice 2 and 1	twice 2 and 1 are 5	2 are contained in 5, twice and 1 remaining
6 are 3 times 2	3 times 2 are 6	2 are contained in 6, 3 times
7 are 3 times 2 and 1	3 times 2 and 1 are 7	2 are contained in 7, 3 times, 1 remaining
And so on.	And so on.	And so on.

Third Line.

3 are once 3	once 3 are 3	3 are contained in 3, once
4 are once 3 and 1	once 3 and 1 are 4	3 are contained in 4, once and 1 remaining
5 are once 3 and 2	once 3 and 2 are 5	3 are contained in 5, once and 2 remaining
6 are twice 3	twice 3 are 6	3 are contained in 6, twice
7 are twice 3 and 1	twice 3 and 1 are 7	3 are contained in 7, twice and 1 remaining
8 are twice 3 and 2	twice 3 and 2 are 8	3 are contained in 8, twice and 2 remaining
And so on.	And so on.	And so on.

Fourth Line.

4 are once 4	once 4 are 4	4 are contained in 4, once
5 are once 4 and 1	once 4 and 1 are 5	4 are contained in 5, once and 1 remaining
6 are once 4 and 2	once 4 and 2 are 6	4 are contained in 6, once and 2 remaining
7 are once 4 and 3	once 4 and 3 are 7	4 are contained in 7, once and 3 remaining
8 are twice 4	twice 4 are 8	4 are contained in 8, twice
9 are twice 4 and 1	twice 4 and 1 are 9	4 are contained in 9, twice and 1 remaining
And so on.	And so on.	And so on.

In the same way the fifth, sixth, seventh, eighth, and ninth lines are to be gone through, terminating with the

Tenth Line.

10 are once 10	once 10 are 10	10 are contained in 10, once
11 are once 10 and 1	once 10 and 1 are 11	10 are contained in 11, once and 1 remaining
12 are once 10 and 2	once 10 and 2 are 12	10 are contained in 12, once and 2 remaining
13 are once 10 and 3	once 10 and 3 are 13	10 are contained in 13, once and 3 remaining
.. .. ..	.. .. ..	.. ..
20 are twice 10	twice 10 are 20	10 are contained in 20, twice
And so on.	And so on.	And so on.

Questions on the Third Exercise.

Second Line.

i. What are 4 times 2 and 1?....*Ans.* Nine.

ii. How many twos can you take out of 11?....*Ans.* Five twos and 1 remaining.

iii. If 11 marbles are distributed among 5 boys, so that each boy may have 2, how many remain?....*Ans.* One remains.

Third Line.

i. Twice 3 and 2 make how many?....*Ans.* Eight.

ii. How many threes are contained in 7?....*Ans.* Two threes and 1 remaining.

iii. If I divide 10 nuts amongst 3 boys, giving 3 nuts to each boy, how many shall I have remaining?....*Ans.* One remaining.

iv. If I have 20 apples, among how many boys can I distribute them, supposing I give 3 to each boy?....*Ans.* Among 6 boys; and 2 apples remaining.

Fourth Line.

i. Three times 4 and 2 are what number?....*Ans.* Fourteen.

ii. How many fours are there contained in 17?....*Ans.* Four fours and 1 remaining.

iii. If 23 loaves be distributed amongst 5 women, how many loaves remain, after giving 4 to each woman?....*Ans.* 3.

iv. How many boys can be supplied with nuts from a bag containing 23 nuts at the rate of 4 to each boy?....*Ans.* Five boys; and 3 nuts remaining.

Similar questions may be given on the fifth, sixth, seventh, eighth, and ninth lines.

Tenth Line.

i. 3 times 10 and 2 are what number?....*Ans.* Thirty-two.

ii. How many tens are contained in 34?....*Ans.* Three tens, and 4 remaining.

iii. If 42 apples be distributed amongst 4 boys, how many apples remain, after giving 10 to each boy?....*Ans.* Two apples.

iv. If I divide 23 apples between 2 boys, how many remain, after giving 10 to each boy?....*Ans.* Three remain.

Questions on Money, &c.

Second Line.

i. What have I remaining out of 11*d*., after paying for 5 oz. of coffee, at 2*d*. per oz.?....*Ans*. 1 penny; because if 1 oz. cost 2*d*., 5 oz. will cost 5 times 2*d*. or 10*d*.; then 11 are 5 times 2 and 1 remaining.

ii. How many pence should I receive in exchange for 7 half-pence?....*Ans*. $3\frac{1}{2}$*d*.; because 2 is contained in 7 three times, and 1 remaining.

Third Line.

i. How much have I remaining out of 10*d*., after paying for 3 oz. of tobacco, at 3*d*. per oz.?....*Ans*. 1 penny; because if 1 oz. cost 3*d*., 3 oz. will cost 3 times 3*d*. or 9*d*.; then 10 are 3 times 3 and 1 remaining.

ii. How many feet are contained in 4 yards 2 feet?.... *Ans*. 14 feet. In 1 yard there are 3 feet; 4 times 3 and 2 are 14.

iii. How many lemons can I buy for 10*d*., supposing each lemon to cost 3*d*.?....*Ans*. 3 lemons; and 1*d*. remaining.

Fourth Line.

i. How many farthings are there in $4\frac{1}{4}$*d*.?....*Ans*. 17; because in a penny there are 4 farthings, and 4 times 4 and 1 are 17.

ii. In 9 farthings how many pence?....*Ans*. $2\frac{1}{4}$*d*.; because 4 is contained in 9 twice and 1 remaining.

iii. How much shall I have left out of 10*d*., after paying for 2 oz. of tea, at 4*d*. per oz.?....*Ans*. 2*d*.

iv. How many fourpenny-pieces are there in 15*d*.?.... *Ans*. 3 pieces and 3*d*. remaining.

Twelfth Line*.

i. How many shillings are there in 19*d*.?....*Ans*. 1*s*. and 7*d*. remaining; because 12 is contained in 19, once and 7 remaining.

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* In the Table of Simple Unity published to accompany this book, the number of lines extends to twelve.

ii. 2*s*. 5*d*. contain how many pence?....*Ans*. 29*d*.; because twice 12 and 5 are 29.

iii. How many pence must I receive for 1*s*. 9*d*.?....*Ans*. 21*d*.

iv. How many pence have I remaining out of 26*d*. after buying 2 lbs. of sugar, at 1*s*. per lb.?....*Ans*. 2*d*.

Miscellaneous Questions on Shillings and Pounds.

i. How many shillings are contained in 1*l*. 7*s*.?....*Ans*. 27*s*.; because, once 20 and 7 are 27.

ii. How many pounds sterling are there in 34*s*.?....*Ans*. 1*l*. and 14*s*. remaining; because 20 are contained in 34 once and 14 remaining.

iii. How many shillings are contained in 1*l*. 14*s*.?.... *Ans*. 34*s*.

iv. How many shillings have I remaining out of 65*s*. after paying 3 workmen 1*l*. each?....*Ans*. 5*s*. remaining.

v. How many yards of cloth can I buy with 2*l*. 14*s*., supposing each yard to cost 20*s*.?....*Ans*. 2 yards, and 14*s*. remaining.

FOURTH EXERCISE.

Division, when the Remainder is expressed as a part of the Divisor.

In the first column of this exercise the remainder in Division is expressed as a part of the divisor: thus, when the divisor is 4 and the dividend 9, we say, "9 are twice 4 and the fourth of 4." The second column gives the reverse operation, "twice 4 and the fourth of 4 are 9."

The teacher in this exercise proceeds in the following manner: Let it be required, for example, to show that "7 are twice 3 and the third of 3;" using the right hand rod, he points off seven units on the third line, then with the left hand

rod he shows that these units contain 2 threes and one over, or that $7 = 2 \times 3 + 1$; but the unit that is here in excess is the third part of 3; therefore the pupils say, "7 are twice 3 and the third of 3," and then conversely, before the pointer is removed, "twice 3 and the third of 3 are 7." Such questions as the following may form part of the exercise:

Teacher. I am going to question you on the line of fours. Here I point to nine marks; how many fours can you take out of 9?....*Pupil.* Two fours and 1 remaining.

Teacher. What part of four is 1?....*Pupil.* One is the fourth of four, so that 9 is made up of 2 fours and the fourth of 4.

Teacher. You now can find out how much twice 4 and the fourth of 4 amount to?....*Pupil.* Nine: because twice 4 are 8, and the fourth of 4 is 1, therefore 8 and 1 are 9.

Second Line, or Line of Twos.

1 is the half of 2	The half of 2 is 1
2 are once 2	Once 2 is 2
3 are once 2 and the half of 2	Once 2 and the half of 2 are 3
4 are twice 2	Twice 2 are 4
5 are twice 2 and the half of 2	Twice 2 and the half of 2 are 5
6 are three times 2	Three times 2 are 6
7 are three times 2 and the half of 2	Three times 2 and the half of 2 are 7
10 are five times 2	Five times 2 are 10

Third Line, or Line of Threes.

1 is the third of 3	The third of 3 is 1
2 are twice the third of 3	Twice the third of 3 are 2
3 are once 3	Three times the third of 3 are 3
4 are once 3 and the third of 3	Once 3 and the third of 3 are 4
5 are once 3 and twice the third of 3	Once 3 and twice the third of 3 are 5
6 are twice 3	Twice 3 are 6
7 are twice 3 and the third of 3	Twice 3 and the third 3 are 7
8 are twice 3 and twice the third of 3	Twice 3 and twice the third of 3 are 8
30 are 10 times 3	Ten times 3 are 30

Fourth Line, or Line of Fours.

1 is the fourth part of 4	The fourth of 4 is 1
2 are twice the fourth of 4	Twice the fourth of 4 are 2
3 are three times the fourth of 4	Three times the fourth of 4 are 3
4 are once 4	Four times the fourth of 4 are 4
5 are once 4 and the fourth of 4	Once 4 and the fourth of 4 are 5
6 are once 4 and twice the fourth of 4	Once 4 and twice the fourth of 4 are 6
7 are once 4 and three times the fourth of 4	Once 4 and three times the fourth of 4 are 7
8 are twice 4	Twice 4 are 8
9 are twice 4 and the fourth of 4	Twice 4 and the fourth of 4 are 9
40 are ten times 4	Ten times 4 are 40

Fifth Line, or Line of Fives.

1 is the fifth part of 5	The fifth part of 5 is 1
2 are twice the fifth of 5	Twice the fifth of 5 are 2
3 are three times the fifth of 5	Three times the fifth of 5 are 3
4 are four times the fifth of 5	Four times the fifth of 5 are 4
5 are once 5	Five times the fifth of 5 are 5
6 are once 5 and the fifth of 5	Once 5 and the fifth of 5 are 6
7 are once 5 and twice the fifth of 5	Once 5 and twice the fifth of 5 are 7
8 are once 5 and three times the fifth of 5	Once 5 and three times the fifth of 5 are 8
50 are ten times 5	Ten times 5 are 50

And so on to the sixth, seventh, eighth, &c., lines.

This table gives us the reason for the operation in Division, when there is a remainder. For example, let it be required to divide 42 by 5, or to find how many fives are contained in 42. It will be seen, on the fifth line, that 42 contains 8 fives, and 2 units over, or 8 fives and 2 fifths of 5; that is, 5 are contained in 42, $8\frac{2}{5}$ times.

Questions on the Fourth Exercise.

Second Line.

i. How many times are 2 contained in 5?....*Ans.* Two and a half times: because twice 2 make 4, and the half of 2 is 1, therefore twice 2 and the half of 2 is the same as 4 and 1, that is, 5.

ii. What are three and a half times 2?....*Ans.* 7: because three times 2 are 6, and the half of 2 is 1; therefore three and a half times 2 are 6 and 1, which are 7.

Third Line.

i. Divide 10 by 3.....*Ans.* 10 contains 3 times 3, and the third of 3; that is, three and a third times.

ii. How many threes are contained in 8?....*Ans.* Twice 3 and twice the third of 3 are 8.

iii. What is 4 times 3 and twice the third of 3?....*Ans.* 14; because 4 times 3 are 12, and twice the third of 3 are 2; therefore, 12 and 2 make 14.

Fourth Line.

i. How often is 4 contained in 21?....*Ans.* Five times and one fourth: because 5 times 4 and the fourth of 4 are 21.

ii. What is 6 times 4 and 3 times the fourth of 4?.... *Ans.* 24 and 3 = 27.

iii. What must I add to 8 times 4 to make up 34?.... *Ans.* 2. Because eight times 4 are 32; 32 and 2 are 34.

iv. If 3 times the fourth of 4 be taken from 5 times 4, what remains?....*Ans.* 17: because 5 times 4 are 20, and 3 times the fourth of 4 are 3; 3 from 20 leaves 17.

v. What remains when twice 4 are taken from 4 times 4 and twice the fourth of 4?....*Ans.* 10. Because twice 4 are 8; 4 times 4 and twice the fourth of 4 are 18; therefore, 8 taken from 18 leaves 10.

Fifth Line.

i. What is the amount of twice 5 and 3 times the fifth of 5?....*Ans.* 13: because twice 5 are 10, and 3 times the fifth of 5 is 3, therefore 10 and 3 make 13.

ii. How many fives can you take out of 11?....*Ans.* Two fives and the fifth of five.

iii. 4 is what part of 5?....*Ans.* 1 is the fifth of 5, and therefore 4 is 4 times the fifth of 5.

iv. What are 4 times 5 and twice the fifth of 5?....*Ans.* 22; because 4 times 5 are 20, and twice the fifth of 5 are 2, and 20 and 2 are 22.

Tenth Line.

i. How many tens are there in 23?....*Ans.* Two tens and 3 times the tenth of 10.

ii. What are 4 times 10 and 9 times the tenth of 10?.... *Ans.* 49.

Second and Third Lines.

i. In 3 times 2 and the half of 2, how many threes?.... *Ans.* 2 threes and the third of 3; because, on the second line we find, that 3 times 2 and the half of 2 are 7; and on the third line it is shewn that 7 are twice 3 and the third of 3.

ii. How many threes are there in 4 times 2 and the half of 2?....*Ans.* 3 threes.

iv. Twice 3, and 3 times 3 and the third of 3, contain how many twos?....*Ans.* 8 twos.

Third and Fourth Lines.

i. How many fours are contained in 5 times 3, and twice the third of 3?....*Ans.* 4 fours and the fourth of 4.

ii. 5 threes and the third of 3 contain how many fours? ....*Ans.* 4 fours.

iii. How many threes can you find in twice 4 and 3 times the fourth of 4?....*Ans.* 3 threes and twice the third of 3.

Fourth and Fifth Lines.

i. Twice 4 and twice the fourth of 4 contain how many fives?....*Ans.* 2 fives.

ii. 3 times 5 and twice the fifth of 5 contain how many fours?....*Ans.* 4 fours and the fourth of 4.

iii. How many fives are contained in 4 times 4 and 3 times the fourth of 4?....*Ans.* 3 fives and 4 times the fifth of 5.

Second and Fourth Lines.

i. 9 times 2 and the half of 2 contain how many fours?.... *Ans.* 4 fours and 3 times the fourth of 4.

ii. In 3 times the fourth of 4, how many twos?....*Ans.* One 2 and the half of 2.

Second and Fifth Lines.

i. 8 times 2 and 4 times the half of 2 contain how many fives?....*Ans.* 4 fives.

ii. In 3 times the fifth of 5 and twice 5 how many twos? ....*Ans.* 6 twos and once the half of 2.

Second and Sixth Lines.

i. 10 times 2 and 6 times the half of 2 are how many sixes?....*Ans.* 4 sixes and twice the sixth of 6.

ii. In 8 times the sixth of 6 and 3 times the half of 2, how many twos?....*Ans.* Five twos and once the half of two.

Similar questions may be proposed on any other two lines.

FIFTH EXERCISE.

Ratios.

In the preceding exercise two numbers are compared, chiefly with the view of ascertaining the result of the division of the one number by the other: in the present exercise is considered the result of such division, or decomposition, as giving the relative magnitude, or *ratio*, of the two numbers. For example, in the course of this exercise it is shown that 9 are 3 times the half of 6; here, then, the ratio of 9 to 6 is 3 times the half, that is, 9 is produced by repeating the half of 6 three times.

As the ratio of two numbers is their relative magnitude, or the number of times that the one number is contained in the other, it follows that this ratio may be given in different forms:

thus the comparison of 12 with 8 may be given in the three following forms:

i. 12 are 12 times the eighth of 8.
ii. 12 are 6 times the fourth of 8.
iii. 12 are 3 times the half of 8.

In the first case the ratio is 12 times the eighth; in the second 6 times the fourth; and in the third 3 times the half. In the last form the ratio is in its least terms, because the numbers expressing it are the smallest that can be used. Hence, it will be observed that 12 times the eighth, 6 times the fourth, and 3 times the half, mean the same thing, or rather, that they indicate operations which produce the same results.

There are three classes of problems which illustrate the nature and use of ratios. These are, 1st. When a number is required which is a given part of a given number; 2nd. When two numbers are given, to find their ratio; and 3rd. When a given number is a given part of a number required. The peculiarity of these three forms of ratios will be best understood by a strict attention to the exercises and questions which accompany them.

Case I. *When a number is required which is a given part of a given number.*

This case may be illustrated by the following question:

What number is 3 times the fourth of 12?....*Ans.* 9.

The teacher calls the attention of the pupils to the first four marks or units on the second line, and then with another rod points to the two units in the first square of that line; the pupils say "the half of 4 is 2." The teacher then removes the latter pointer to the third square, and the pupils say, "3 times the half of 4 are 3 times 2, or 6." Still keeping the first pointer in its place, the teacher places the other on the fourth square, when the pupils say "4 times the half of 4 are 4 times 2, or 8," and so on. Questions similar to the following may be put in the course of the exercise.

What do you understand by 4 times the half of 4?....*Ans.* That the half of 4, which is 2, is to be taken four times.

On what line do you find 7 times the third of 12?....*Ans.* On the fourth line 12 units are distributed in three squares, which shows that the third of 12 is 4, and seven squares from this line give me 7 times 4, which are 28.

First Line.

The half of 2 is 1.
Twice the half of 2 are twice 1, or 2.
3 times the half of 2 are 3 times 1, or 3.
4 times the half of 2 are 4 times 1, or 4.
And so on to
10 times the half of 2 are 10 times 1, or 10.

The third of 3 is 1.
Twice the third of 3 are twice 1, or 2.
3 times the third of 3 are 3 times 1, or 3.
4 times the third of 3, are 4 times 1, or 4.
And so on to
10 times the third of 3 are 10 times 1, or 10.

And so on, until the teacher concludes the line with

The tenth of 10 is 1.
Twice the tenth of 10 are twice 1, or 2.
3 times the tenth of 10 are 3 times 1, or 3.
4 times the tenth of 10 are 4 times 1, or 4.
And so on to
10 times the tenth of 10 are 10 times 1, or 10.

Second Line.

The half of 4 is 2.
3 times the half of 4 are 3 times 2, or 6.
4 times the half of 4 are 4 times 2, or 8.
5 times the half of 4 are 5 times 2, or 10.
And so on to
10 times the half of 4 are 10 times 2, or 20.

The third part of 6 is 2.
Twice the third of 6 are twice 2, or 4.
4 times the third of 6 are 4 times 2, or 8.
5 times the third of 6 are 5 times 2, or 10.
And so on to
10 times the third of 6 are 10 times 2, or 20.

The fourth part of 8 is 2.
Twice the fourth of 8 are twice 2, or 4.
3 times the fourth of 8 are 3 times 2, or 6.
5 times the fourth of 8 are 5 times 2, or 10.
And so on to
10 times the fourth of 8 are 10 times 2, or 20.

The teacher proceeds in this manner, taking successively the fifth of 10, the sixth of 12, the seventh of 14, the eighth of 16, the ninth of 18, concluding with

The tenth of 20 is 2.
Twice the tenth of 20 are twice 2, or 4.
3 times the tenth of 20 are 3 times 2, or 6.
4 times the tenth of 20 are 4 times 2, or 8.
5 times the tenth of 20 are 5 times 2, or 10.
And so on to
9 times the tenth of 20 are 9 times 2, or 18.

Third Line.

The half of 6 is 3.
3 times the half of 6 are 3 times 3, or 9.
4 times the half of 6 are 4 times 3, or 12.
5 times the half of 6 are 5 times 3, or 15.
6 times the half of 6 are 6 times 3, or 18.
And so on to
10 times the half of 6 are 10 times 3, or 30.

The third of 9 is 3.
Twice the third of 9 are twice 3, or 6.
4 times the third of 9 are 4 times 3, or 12.
5 times the third of 9 are 5 times 3, or 15.
6 times the third of 9 are 6 times 3, or 18.
And so on to
10 times the third of 9 are ten times 3, or 30.

And so on, concluding with

The tenth of 30 is 3.
Twice the tenth of 30 are twice 3, or 6.
3 times the tenth of 30 are 3 times 3, or 9.
4 times the tenth of 30 are 4 times 3, or 12.
5 times the tenth of 30 are 5 times 3, or 15.
And so on to
9 times the tenth of 30 are 9 times 3, or 27.

Fourth Line.

The half of 8 is 4.
3 times the half of 8 are 3 times 4, or 12.
4 times the half of 8 are 4 times 4, or 16.
5 times the half of 8 are 5 times 4, or 20.
6 times the half of 8 are 6 times 4, or 24.
And so on to
10 times the half of 8 are 10 times 4, or 40.

The third of 12 is 4.
Twice the third of 12 are twice 4, or 8.
4 times the third of 12 are 4 times 4, or 16.
5 times the third of 12 are 5 times 4, or 20.
6 times the third of 12 are 6 times 4, or 24.
And so on to
10 times the third of 12 are 10 times 4, or 40.

Proceeding in this way through the various multiples of 4, we conclude with

The tenth of 40 is 4.
Twice the tenth of 40 are twice 4, or 8.
3 times the tenth of 40 are 3 times 4, or 12.
4 times the tenth of 40 are 4 times 4, or 16.
5 times the tenth of 40 are 5 times 4, or 20.
And so on to
9 times the tenth of 40 are 9 times 4, or 36.

And so on to the 5th, 6th, 7th, &c. lines, concluding with the

Tenth Line.

The half of 20 is 10.
3 times the half of 20 are 3 times 10, or 30.
4 times the half of 20 are 4 times 10, or 40.
5 times the half of 20 are 5 times 10, or 50.
And so on to
10 times the half of 20 are 10 times 10, or 100.

Proceeding on this way through the different compartments we come to the ninth compartment of this line.

The ninth of 90 is 10.
Twice the ninth of 90 are twice 10, or 20.
3 times the ninth of 90 are 3 times 10, or 30.
4 times the ninth of 90 are 4 times 10, or 40.
And so on to
10 times the ninth of 90 are 10 times 10, or 100.

The tenth of 100 is 10.
Twice the tenth of 100 are twice 10 or 20.
3 times the tenth of 100 are 3 times 10, or 30.
4 times the tenth of 100 are 4 times 10, or 40.
And so on to
9 times the tenth of 100 are 9 times 10, or 90.

The teacher may then propose examples relating to various lines and squares promiscuously: thus,

4 times the fifth of 45 are 4 times 9, or 36.
5 times the seventh of 35 are 5 times 5, or 25.
3 times the sixth of 24 are 3 times 4, or 12.
Twice the fifth of 35 are twice 7, or 14.
&c. &c.

Questions on Case I.

First Line.

i. What is 3 times the half of 2?....*Ans.* 3.

Proof. The half of 2 is 1, and 3 times the half of 2 are 3 times 1, or 3.

ii. What is 5 times the fourth of 4?....*Ans.* 5.

Proof. The fourth of 4 is 1, and 5 times the fourth of 4 are 5 times 1, or 5.

Second Line.

i. What is 5 times the half of 4?....*Ans.* 10.

Proof. The half of 4 is 2, and 5 times the half of 4 are 5 times 2, or 10.

ii. What is 7 times the fifth of 10?....*Ans.* 14.

Proof. The fifth of 10 is 2, and 7 times the fifth of 10 are 7 times 2, or 14.

Third Line.

i. What is 5 times the half of 6?....*Ans.* 15.

Proof. The half of 6 is 3, and 5 times the half of 6 are 5 times 3, or 15.

ii. 3 times the fifth of 15 is what number?....*Ans.* 9.

Proof. The fifth of 15 is 3, and 3 times the fifth of 15 are 3 times 3, or 9.

iii. What is 8 times the sixth of 18?....*Ans.* 24.

Proof. The sixth of 18 is 3, and 8 times the sixth of 18 are 8 times 3, or 24.

Fourth Line.

i. What is 9 times the third of 12?....*Ans.* 36.

Proof. The third of 12 is 4, and 9 times the third of 12 are 9 times 4, or 36.

ii. What is 6 times the ninth of 36?....*Ans*. 24.
Proof. The ninth of 36 is 4, and 6 times the ninth of 36 are 6 times 4, or 24.

iii. 5 times the sixth of 24 is what number?....*Ans*. 20.
Proof. The sixth of 24 is 4, and 5 times the sixth of 24 are 5 times 4, or 20.

Fifth Line.

i. What is 4 times the sixth of 30?....*Ans*. 20.
Proof. The sixth of 30 is 5, and 4 times the sixth of 30 are 4 times 5, or 20.

ii. What number is 8 times the seventh of 35?....*Ans*. 40.
Proof. The seventh of 35 is 5, and 8 times the seventh of 35 are 8 times 5, or 40.

iii. 9 times the fifth of 25 is what number?....*Ans*. 45.
Proof. The fifth of 25 is 5, and 9 times the fifth of 25 are 9 times 5, or 45.

The Lines taken promiscuously.

i. What number is 7 times the sixth of 12?....*Ans*. 14.
Proof. The sixth of 12 is 2, and 7 times the sixth of 12 are 7 times 2, or 14.

ii. What is 3 times the fifth of 20?....*Ans*. 12.
Proof. The fifth of 20 is 4, and 3 times the fifth of 20 are 3 times 4, or 12.

iii. 9 times the seventh of 42 is what number?....*Ans*. 54.
Proof. The seventh of 42 is 6, and 9 times the seventh of 42 are 9 times 6, or 54.

iv. What number is 8 times the ninth of 63?....*Ans*. 56.
Proof. The ninth of 63 is 7, and 8 times the ninth of 63 are 8 times 7, or 56.

v. What is 5 times the tenth of 90?....*Ans*. 45.
Proof. The tenth of 90 is 9, and 5 times the tenth of 90 are 5 times 9, or 45.

Questions on Money, Weights, and Measures.

First Line.

i. What is the amount of 3 times the half of 2*s.*?....*Ans.* 3*s.*

Proof. The half of 2*s.* is 1*s.*, and 3 times the half of 2*s.* are 3 times 1*s.*, or 3*s.*

ii. What is the amount of 5 times the seventh of 7*d.*?.... *Ans.*. 5*d.*

Proof. The seventh of 7*d.* is 1*d.*, and 5 times the seventh of 7*d.* are 5 times 1*d.*, or 5*d.*

iii. 5 times the ninth of 9 oz. is what number of ounces? ....*Ans.* 5 oz.

Proof. The ninth of 9 oz. is 1 oz., and 5 times the ninth of 9 oz. are 5 times 1 oz., or 5 oz.

iv. If 15 lbs. cost 1*s.* 3*d.*, what is the cost of 7 lbs.?.... *Ans.* 7*d.*

Proof. In 1*s.* 3*d.* there are 15*d.* If 15 lbs. cost 1*s.* 3*d.* 1 lb. will cost 1*d.*, and 7 lbs. will cost 7*d.*

Second Line.

i. What is 5 times the half of 4*d.*?....*Ans.* 10*d.*

Proof. The half of 4*d.* is 2*d.*, and 5 times the half of 4*d.* are 5 times 2*d.*, or 10*d.*

ii. What is 7 times the half of 4 oz.?....*Ans.* 14 oz.

Proof. The half of 4 oz. is 2 oz., and 7 times the half of 4 oz. are 7 times 2 oz., or 14 oz.

iii. What is the amount of 9 times the third of 6*d.*?.... *Ans.* 18*d.*

Proof. The third of 6*d.* is 2*d.*, and 9 times the third of 6*d.* are 9 times 2*d.*, or 18*d.*

iv. What is the amount of 9 times the fourth of 8*d.*? ....*Ans.* 1*s.* 6*d.*

Proof. The fourth of 8*d.* is 2*d.*, and 9 times the fourth of 8*d.* are 9 times 2*d.*, or 18*d.* = 1*s.* 6*d.*

v. If 6 lbs. of coffee cost 12*s.* what is the price of 8 lbs.? ....*Ans.* 16*s.*

Proof. If 6 lbs. cost 12*s.* 1 lb. will cost 2*s.*, and 8 lbs. will cost 8 times 2*s.*, or 16*s.*

Third Line.

i. What is 6 times the fifth of 1*s*. 3*d*.?....*Ans*. 1*s*. 6*d*.

Proof. The fifth of 1*s*. 3*d*. is 3*d*., and 6 times the fifth of 1*s*. 3*d*. are 6 times 3*d*., or 1*s*. 6*d*.

ii. If 1 lb. of tea cost 3*s*. 3*d*. what is the cost of 6 times the thirteenth part of 1 lb.?....*Ans*. 18*d*.

Proof. The cost of the thirteenth part of 1 lb. must be the thirteenth part of 3*s*. 3*d*., or 3*d*., and 6 times the thirteenth of 3*s*. 3*d*., are 6 times 3*d*. or 18*d*.

iii. If 1 lb. of sugar cost 9*d*. what is the amount of 4 times the third of 1 lb.?....*Ans*. 1*s*.

Proof. The cost of 1 lb. is 9*d*.; the cost of the third of 1 lb. is the third of 9*d*., or 3*d*., and 4 times the third of 9*d*. are 4 times 3*d*., or 1*s*.

iv. If 6 lbs. of tobacco cost 18*s*. what is the price of 5 lbs.? ....*Ans*. 15*s*.

Proof. If 6 lbs. cost 18*s*. 1 lb. will cost 3*s*., and 5 lbs. will cost 5 times 3*s*., or 15*s*.

v. What is the value of 3 times the seventh of a guinea? ....*Ans*. 9*s*.

Proof. The seventh of a guinea is 3*s*., and 3 times the seventh are 3 times 3*s*., or 9*s*.

vi. What is the amount of 3 times the fifteenth of 3*s*. 9*d*.?....*Ans*. 9*d*.

Proof. The fifteenth of 3*s*. 9*d*. is 3*d*., and 3 times the fifteenth of 3*s*. 9*d*. are 3 times 3*d*., or 9*d*.

vii. What is the amount of 3 times the sixth of 1*s*. 6*d*.? ....*Ans*. 9*d*.

Proof. The sixth of 1*s*. 6*d*. is 3*d*., and 3 times the sixth of 1*s*. 6*d*. are 3 times 3*d*., or 9*d*.

viii. What is the amount of 4 times the fifth of 1*s*. 3*d*.? ....*Ans*. 1*s*.

Proof. The fifth of 1*s*. 3*d*. is 3*d*., and 4 times the fifth of 1*s*. 3*d*. are 4 times 3*d*., or 1*s*.

Fourth Line.

i. Divide 28*l*. equally amongst 7 persons.....*Ans*. Each would receive 4*l*.

Proof. If 7 persons receive 28*l*. one person would receive the seventh part of 28*l*., or 4*l*.

ii. Divide 32*l.* between 2 persons, so that one shall have 3 times the eighth of the money, and the other the remaining part.....*Ans.* One would receive 12*l.* and the other 20*l.*

Proof. The eighth of 32*l.* is 4*l.*, and 3 times the eighth of 32*l.* are 3 times 4*l.*, or 12*l.*; then 32*l.* less by 12*l.* = 20*l.*

The Lines taken promiscuously.

i. What is the amount of 3 times the fifth of 1*s.* 8*d.*?.... *Ans.* 1*s.*

Proof. 1*s.* 8*d.* are 20*d.*, the fifth of 20*d.* is 4*d.*, and 3 times the fifth of 20*d.*, or 3 times 4*d.*, or 12*d.* = 1*s.*

ii. How much is 8 times the seventh of 5*s.* 3*d.*?....*Ans.* 6*s.*

Proof. The seventh of 63*d.* is 9*d.*, and 8 times the seventh of 63*d.* are 8 times 9*d.*, or 72*d.* = 6*s.*

iii. How many pounds are there in 8 times the seventh of 63 oz.?....*Ans.* 4 lbs. 8 oz.

Proof. The seventh of 63 oz. is 9 oz., and 8 times the seventh of 63 oz. are 8 times 9 oz., or 72 oz.=4 lb. 8 oz.

iv. What is 9 times the third of 3 quarters of a yard? ....*Ans.* $2\frac{1}{4}$ yds.

Proof. The third of 3 qrs. is 1 qr.; therefore 9 times the third of 3 qrs. is 9 times 1 qr., or 9 qrs.=$2\frac{1}{4}$ yds.

v. What is 7 times the sixth of 3*s.* 6*d.*?....*Ans.* 4*s.* 1*d.*

Proof. 3*s.* 6*d.* are 42*d.*; the sixth of 42*d.* is 7*d.*, and 7 times the sixth of 42*d.* are 7 times 7, or 49*d.*=4*s.* 1*d.*

vi. If 8 oz. of tea cost 2*s.* 8*d.*, what is the price of 9 oz.? ....*Ans.* 3*s.*

Proof. In 2*s.* 8*d.* there are 32 pence. If 8 oz. cost 32 pence, 1 oz. will cost 4*d.*, and 9 oz. will cost 9 times 4, or 36 pence. 36 pence are 3*s.*

vii. What is the value of 10 yards of calico, if 4 yards cost 1*s.* 4*d.*?....*Ans.* 3*s.* 4*d.*

Proof. In 1*s.* 4*d.* there are 16 pence. If 4 yards cost 16 pence, 1 yard will cost 4 pence, and 10 yards will cost 10 times 4, or 40 pence. 40 pence are 3*s.* 4*d.*

Case II. *Where two numbers are given, to find their ratio.*

This case may be illustrated by the following question: What is the ratio between 9 and 12?

The left hand pointer is to be placed on the third square of the third line, and the pupils say, "9 are 3 times 3;" the right hand pointer is then to be placed on the second square, and the pupils say, "6 are twice 3, 3 times 3 are 3 times the half of twice 3." The teacher now removes the right hand pointer to the fourth square, and the pupils repeat the next step, "12 are 4 times 3," "3 times 3 are 3 times the fourth of 4 times 3;" and so on, always observing to keep the first pointer in its place. When two numbers are to be compared, we seek to discover their lowest ratio, by finding the highest number which will divide them both. This number gives on the unity board the line from which the ratio is to be taken. For example, let it be required to exhibit the ratio of 12 to 8. On the fourth line, the teacher points with the right hand rod to 12 units or marks which the pupil observes are made up of 3 fours; then with the left hand rod, he points off 8 units or marks, which are made up of 2 fours; hence, 12 is 3 times the half of 8, because the half of 8, which is 4, taken 3 times produces 12. By this means we obtain the ratio of 12 to 8, in its lowest terms. As another example, let the ratio of 6 to 9 be required. A line is sought for upon the board, where 6 and 9 are made up of similar groups of units; this is readily found to be the third line; then according to the language of the exercise we have

6 are twice 3, 9 are 3 times 3.
Twice 3 are twice the third of 3 times 3,
that is, 6 are twice the third of 9.

In some cases the teacher may find it necessary, at every successive ratio, to repeat the decomposition given at the head of each table, as in the following example:

6 are 3 times 2.
4 are twice 2. 3 times 2 are 3 times the half of twice 2.
6 are 3 times 2.
8 are 4 times 2. 3 times 2 are 3 times the fourth of 4 times 2.
6 are 3 times 2.
10 are 5 times 2. 3 times 2 are 3 times the fifth of 5 times 2.

It may here be stated, once for all, that whenever two or more numbers are made the subject of comparison, the teacher should use two pointers; for by this means, the objects of comparison are kept before the eye of the pupil.

As a great proportion of the questions which arise in the business of life involve the principle of ratios, the present exercise should be thoroughly understood.

First Line.

1 is the half of twice 1 or 2.
1 is the third of 3 times 1 or 3.
1 is the fourth of 4 times 1 or 4.
1 is the fifth of 5 times 1 or 5.
1 is the tenth of 10 times 1 or 10.

2 are twice 1, or twice the third of 3 times 1 or 3.
2 are twice 1, or twice the fourth of 4 times 1 or 4.
2 are twice 1, or twice the fifth of 5 times 1 or 5.
2 are twice 1, or twice the tenth of 10 times 1 or 10.

3 are 3 times 1, or 3 times the half of twice 1 or 2.
3 are 3 times 1, or 3 times the third of 3 times 1 or 3.
3 are 3 times 1, or 3 times the fourth of 4 times 1 or 4.
3 are 3 times 1, or 3 times the tenth of 10 times 1 or 10.
And so on to the ratios of 4, 5, 6, &c.

Second Line, or Line of Twos.

On this line 2 is first compared with the multiples of itself; then 4, 6, 8, &c. are successively compared with *all* the multiples of 2 as far as 10 times 2.

4 are twice 2.	2 are the half of twice 2 or 4.
6 are 3 times 2.	2 are the third of 3 times 2 or 6.
8 are 4 times 2.	2 are the fourth of 4 times 2 or 8.
And so on.	

4 are twice 2.	Twice 2 are twice the half of twice 2.
6 are 3 times 2.	Twice 2 are twice the third of 3 times 2.
8 are 4 times 2.	Twice 2 are twice the fourth of 4 times 2.
10 are 5 times 2.	Twice 2 are twice the fifth of 5 times 2.
And so on.	

6 are 3 times 2.

4 are twice 2.	3 times 2 are 3 times the half of twice 2.
8 are 4 times 2.	3 times 2 are 3 times the fourth of 4 times 2.
10 are 5 times 2.	3 times 2 are 3 times the fifth of 5 times 2.
And so on.	

8 are 4 times 2.

4 are twice 2.	4 times 2 are 4 times the half of twice 2.
6 are 3 times 2.	4 times 2 are 4 times the third of 3 times 2.
8 are 4 times 2.	4 times 2 are 4 times the fourth of 4 times 2.
10 are 5 times 2.	4 times 2 are 4 times the fifth of 5 times 2.
And so on.	

10 are 5 times 2.

4 are twice 2.	5 times 2 are 5 times the half of twice 2.
6 are 3 times 2.	5 times 2 are 5 times the third of 3 times 2.
8 are 4 times 2.	5 times 2 are 5 times the fourth of 4 times 2.
10 are 5 times 2.	5 times 2 are 5 times the fifth of 5 times 2.

And so on.

The teacher then goes on to compare 6 times 2, 7 times 2, 8 times 2, &c., with all the multiples of 2.

Third Line, or Line of Threes.

6 are twice 3.	3 are the half of twice 3, or 6.
9 are 3 times 3.	3 are the third of 3 times 3, or 9.
12 are 4 times 3.	3 are the fourth of 4 times 3, or 12.
15 are 5 times 3.	3 are the 5th of 5 times 3, or 15.

And so on.

6 are twice 3.	Twice 3 are twice the half of twice 3.
9 are 3 times 3.	Twice 3 are twice the third of 3 times 3.
12 are 4 times 3.	Twice 3 are twice the fourth of 4 times 3.
15 are 5 times 3.	Twice 3 are twice the fifth of 5 times 3.

And so on.

9 are 3 times 3.

6 are twice 3.	3 times 3 are 3 times the half of twice 3.
9 are 3 times 3.	3 times 3 are 3 times the third of 3 times 3.
12 are 4 times 3.	3 times 3 are 3 times the fourth of 4 times 3.
15 are 5 times 3.	3 times 3 are 3 times the fifth of 5 times 3.

And so on.

12 are 4 times 3.

6 are twice 3.	4 times 3 are 4 times the half of twice 3.
9 are 3 times 3.	4 times 3 are 4 times the third of 3 times 3.
12 are 4 times 3.	4 times 3 are 4 times the fourth of 4 times 3.
15 are 5 times 3.	4 times 3 are 4 times the fifth of 5 times 3.

And so on.

15 are 5 times 3.

6 are twice 3.	5 times 3 are 5 times the half of twice 3.
9 are 3 times 3.	5 times 3 are 5 times the third of 3 times 3.
12 are 4 times 3.	5 times 3 are 5 times the fourth of 4 times 3.
15 are 5 times 3.	5 times 3 are 5 times the fifth of 5 times 3.

And so on.

The teacher then goes on to compare 6 times 3, 7 times 3, 8 times 3, &c., with all the multiples of 3.

Fourth Line, or Line of Fours.

8 are twice 4.	4 are the half of twice 4 or 8.
12 are 3 times 4.	4 are the third of 3 times 4 or 12.
16 are 4 times 4.	4 are the fourth of 4 times 4 or 16.
20 are 5 times 4.	4 are the fifth of 5 times 4 or 20.

And so on.

8 are twice 4.	twice 4 are twice the half of twice 4.
12 are 3 times 4.	twice 4 are twice the half of 3 times 4.
16 are 4 times 4.	twice 4 are twice the fourth of 4 times 4.
20 are 5 times 4.	twice 4 are twice the fifth of 5 times 4.

And so on.

12 are 3 times 4.

8 are twice 4.	3 times 4 are 3 times the half of twice 4.
12 are 3 times 4.	3 times 4 are 3 times the third of 3 times 4.
16 are 4 times 4.	3 times 4 are 3 times the fourth of 4 times 4.
20 are 5 times 4.	3 times 4 are 3 times the fifth of 5 times 4.

And so on.

16 are 4 times 4.

8 are twice 4.	4 times 4 are 4 times the half of twice 4.
12 are 3 times 4.	4 times 4 are 4 times the third of 3 times 4.
16 are 4 times 4.	4 times 4 are 4 times the fourth of 4 times 4.
20 are 5 times 4.	4 times 4 are 4 times the fifth of 5 times 4.

And so on.

20 are 5 times 4.

8 are twice 4.	5 times 4 are 5 times the half of twice 4.
12 are 3 times 4.	5 times 4 are 5 times the third of 3 times 4.
16 are 4 times 4.	5 times 4 are 5 times the fourth of 4 times 4.
20 are 5 times 4.	5 times 4 are 5 times the fifth of 5 times 4.

And so on.

The teacher then goes on to compare 6 times 4, 7 times 4, 8 times 4, &c., with all the multiples of 4.

Fifth Line, or Line of Fives.

10 are twice 5.	5 are the half of twice 5 or 10.
15 are 3 times 5.	5 are the third of three times 5 or 15.
20 are 4 times 5.	5 are the fourth of 4 times 5 or 20.

And so on.

10 are twice 5.	Twice 5 are twice the half of twice 5.
15 are 3 times 5.	Twice 5 are twice the third of 3 times 5.
20 are 4 times 5.	Twice 5 are twice the fourth of 4 times 5.
25 are 5 times 5.	Twice 5 are twice the fifth of 5 times 5.

And so on.

15 are 3 times 5.

10 are twice 5.	3 times 5 are 3 times the half of twice 5.
15 are 3 times 5.	3 times 5 are 3 times the third of 3 times 5.
20 are 4 times 5.	3 times 5 are 3 times the fourth of 4 times 5.
25 are 5 times 5.	3 times 5 are 3 times the fifth of 5 times 5.

20 are 4 times 5.

10 are twice 5.	4 times 5 are 4 times the half of twice 5.
15 are 3 times 5.	4 times 5 are 4 times the third of 3 times 5.
20 are 4 times 5.	4 times 5 are 4 times the fourth of 4 times 5.
25 are 5 times 5.	4 times 5 are 4 times the fifth of 5 times 5.

And so on.

And so on to the comparison of 5 times 5, 6 times 5, 7 times 5, &c., with all the multiples of 5.

The form of the exercise being exhibited in these five lines, the teacher will be enabled to extend the same form to the other lines upon the board.

Questions on Case II.

First Line.

i. What is the ratio of 9 to 2?....*Ans.* 9 times the half.

Proof. 9 are 9 times 1, or 9 times the half of twice 1 or 2.

ii. Compare the numbers 3 and 2.....*Ans.* 3 are 3 times the half of 2.

Proof. 3 are 3 times 1, or 3 times the half of twice 1 or 2.

Second Line.

i. What part of 10 is 4?....*Ans.* Twice the fifth.

Proof. 4 are twice 2, 10 are 5 times 2; twice 2 are twice the fifth of 5 times 2.

ii. What part of 14 is 6?....*Ans.* 3 times the seventh.

Proof. 6 are 3 times 2, 14 are 7 times 2; 3 times 2 are 3 times the seventh of 7 times 2.

iii. What is the ratio of 18 to 8?....*Ans.* 9 times the fourth.

Proof. 18 are 9 times 2, 8 are 4 times 2; 9 times 2 are 9 times the fourth of 4 times 2.

Third Line.

i. What part of 18 is 12?....*Ans.* 4 times the sixth part.

Proof. 12 are 4 times 3, 18 are 6 times 3; 4 times 3 are 4 times the sixth of 6 times 3.

ii. What is the ratio of 21 to 15?....*Ans.* 7 times the fifth.

Proof. 21 are 7 times 3, 15 are 5 times 3; 7 times 3 are 7 times the fifth of 5 times 3.

Fourth Line.

i. What part of 20 is 16?....*Ans.* 4 times the fifth.

Proof. 16 are 4 times 4, 20 are 5 times 4; 4 times 4 are 4 times the fifth of 5 times 4.

ii. What part of 28 is 20?....*Ans.* 5 times the seventh.

Proof. 20 are 5 times 4, 28 are 7 times 4; 5 times 4 are 5 times the seventh of 7 times 4.

iii. What part of 8 is 12?....*Ans.* 12 is 3 times the half of 8.

Proof. 12 are 3 times 4, 8 are twice 4, and 3 times 4 are 3 times the half of twice 4.

iv. What is the ratio of 16 to 28?....*Ans.* 16 are 4 times the seventh of 28.

Proof. 16 are 4 times 4; 28 are 7 times 4, and 4 times 4 are 4 times the seventh of 7 times 4.

Fifth Line.

i. Compare 20 with 25. ....*Ans.* 20 are 4 times the fifth of 25.

Proof. 20 are 4 times 5, 25 are 5 times 5, 4 times 5 are 4 times the fifth of 5 times 5.

ii. Compare 15 with 10.....*Ans.* 15 are 3 times the half of 10.

Proof. 10 are twice 5, 15 are 3 times 5, 3 times 5 are 3 times the half of twice 5.

iii. Compare 35 with 30.....*Ans.* 7 times the sixth of 30.

Proof. 35 are 7 times 5, 30 are 6 times 5, 7 times 5 are 7 times the sixth of 6 times 5.

iv. Compare 40 with 45.....*Ans.* 40 are 8 times the ninth of 45.

Proof. 40 are 8 times 5, 45 are 9 times 5, 8 times 5 are 8 times the ninth of 9 times 5.

v. Compare 30 with 45.....*Ans.* 30 are 6 times the ninth of 45.

Proof. 30 are 6 times 5, 45 are 9 times 5, 6 times 5 are 6 times the ninth of 9 times 5.

Questions on Money, Weights, and Measures.

Hitherto the weights and measures, made use of in the questions, have been of the most simple kind; but upon proposing the following questions, the teacher will find it necessary to instruct his pupils in the more common and useful tables.

It is assumed that the teacher has varied the foregoing exercises, by teaching the pupils those rules and operations of common arithmetic, which may be based upon the principles developed in them. At the same time it must be distinctly understood, that the operations contained in these exercises are always intended to precede the corresponding calculations conducted by the use of figures.

In proposing questions which contain three or more quantities, the teacher may write down the quantities upon the black board, while the pupils proceed to work out the result mentally as usual. It may be advisable to defer the more difficult questions, particularly those of a miscellaneous kind, until the pupils have made some further progress as well in the Exercises on Fractions, as in the remaining parts of the present chapter.

First Line.

i. How many times 8*d.* are 9*d.* ?....*Ans.* 9*d.* are 9 times the eighth of 8*d.*

Proof. 9 are 9 times 1, or 9 times the eighth of 8 times 1.

ii. How many times 9*d.* is 11*d.* ?....*Ans.* 11*d.* are 11 times the ninth of 9*d.*

Proof. 11 are 11 times 1, or 11 times the ninth of 9 times 1.

iii. If 7 articles cost 1*s.* 9*d.*, what will be the cost of 5 at the same rate ?....*Ans.* 5 times the seventh of 1*s.* 9*d.* or 1*s.* 3*d.*

Proof. 5 are 5 times the seventh of 7, the cost of 7 is 21*d.*, therefore, the cost of 5 will be 5 times the seventh of the cost of 7 articles, which is 1*s.* 3*d.*

Second Line.

i. If 6 articles cost 9*d.*, what will 10 cost?....*Ans.* 15*d.*

Proof. 10 are 5 times 2, 6 are 3 times 2, 5 times 2 are 5 times the third of 3 times 2. The cost of 6 articles is 9*d.*;

therefore the cost of 10, which are 5 times the third of 6, will be 5 times the third of *9d.* or *15d.*

ii. 8 articles cost 1*s.*, required the value of 14 articles at the same rate.....*Ans.* 1*s.* 9*d.*

Proof. 14 are 7 times 2, 8 are 4 times 2, 7 times 2 are 7 times the fourth of 4 times 2. The cost of 8 articles is 1*s.*, therefore the cost of 14 articles, which are 7 times the fourth of 8, will be 7 times the fourth of 1*s.*, or 21*d.* = 1*s.* 9*d.*

iii. If 10 articles cost 1*s.* 3*d.*, what will 12 articles cost? .....*Ans.* 6 times the fifth of 1*s.* 3*d.* or 1*s.* 6*d.*

Proof. 12 are 6 times 2, 10 are 5 times 2, 6 times 2 are 6 times the fifth of 5 times 2; therefore if 10 articles cost 1*s.* 3*d.*, 12 articles, which are 6 times the fifth of 10, will cost 6 times the fifth of 1*s.* 3*d.* or 18*d.* = 1*s.* 6*d.*

Third Line.

i. If 9 pints of ale cost 2*s.* 3*d.*, what is the worth of 12 quarts?.....*Ans.* 8 times the third of 2*s.* 3*d.* or 6*s.*

Proof. 12 quarts are 24 pints, 24 are 8 times 3, 9 are 3 times 3, 8 times 3 are 8 times the third of 3 times 3, therefore if the cost of 9 pints be 2*s.* 3*d.*, the cost of 24 pints, which are 8 times the third of 9 pints, will be 8 times the third of 2*s.* 3*d.* or 6*s.*

ii. How much will 15 articles cost, if the price of 12 be 2*s.* 8*d.*?.....*Ans.* 5 times the fourth of 2*s.* 8*d.* or 3*s.* 4*d.*

Proof. 15 are 5 times 3, 12 are 4 times 3, 5 times 3 are 5 times the fourth of 4 times 3; therefore the cost of 15 articles, which are 5 times the fourth of 12 articles, will be 5 times the fourth of 2*s.* 8*d.* or 3*s.* 4*d.*

iii. What part of a shilling is 9*d.*?.....*Ans.* 3 times the fourth.

Proof. In a shilling there are 12 pence; 9 are 3 times 3, 12 are 4 times 3, 3 times 3 are 3 times the fourth of 4 times 3.

iv. 6*s.* are what part of a guinea?.....*Ans.* Twice the seventh.

Proof. 6 are twice 3, 21 are 7 times 3, twice 3 are twice the seventh of 7 times 3.

v. A man earns 26*s.* per week of 6 days, what are his wages for 9 days?.....*Ans.* 1*l.* 19*s.*

Proof. 9 are 3 times the half of 6; therefore the wages for 9 days will be 3 times the half of 26*s.*, that is, 39*s.*

vi. The gross load of a horse is 6000 lbs., the weight of the waggon is 2000 lbs., what part of the gross load is the weight of the material ?....*Ans.* Twice the third.

Proof. The net load is 4000 lbs.; 4 are twice the third of 6; therefore 4000 is twice the third of 6000.

Fourth Line.

i. If 16 articles be worth 3*s.* 4*d.*, what is the value of 20 articles ?....*Ans.* 20 will cost 5 times the fourth of 3*s.* 4*d.* or 4*s.* 2*d.*

Proof. 20 are 5 times 4, 16 are 4 times 4, 5 times 4 are 5 times the fourth of 4 times 4; therefore if the cost of 16 be 3*s.* 4*d.*, the cost of 20, which are 5 times the fourth of 16, will be 5 times the fourth of 3*s.* 4*d.* or 4*s.* 2*d.*

ii. What part of a pound is 8*s.*?....*Ans.* Twice the fifth.

Proof. 8 are twice 4, 20 are 5 times 4, twice 4 are twice the fifth of 5 times 4.

iii. If a barrel of beer cost 3*l.* 12*s.*, what will 16 gallons cost ?....*Ans.* 4 times the ninth of 3*l.* 12*s.* or 1*l.* 12*s*,

Proof. In a barrel there are 36 gallons, 16 are 4 times 4, 36 are 9 times 4, 4 times 4 are 4 times the ninth of 9 times 4, therefore, if the cost of 36 gallons be 3*l.* 12*s.*, the cost of 16, which are 4 times the ninth of 36 gallons, will be 4 times the ninth of 3*l.* 12*s.* or 1*l.* 12*s.*

Fifth Line.

i. If the cost of 15 articles amount to 2*s.*, what is the value of 40 articles ?....*Ans.* 8 times the third of 2*s.* or 5*s.* 4*d.*

Proof. 40 are 8 times 5, 15 are 3 times 5, 8 times 5 are 8 times the third of 3 times 5; therefore, if the cost of 15 articles be 2*s.*, the cost of 40, which are 8 times the third of 15 articles, will be 8 times the third of 2*s.* or 5*s.* 4*d.*

ii. What is the cost of 3 gallons 1 pint, if the cost of 1 gallon 2 pints be 2*s.* 6*d.*?....*Ans.* 5 times the half of 2*s.* 6*d.* or 6*s.* 3*d.*

Proof. 3 gallons 1 pint are 25 pints, 1 gallon and 2 pints are 10 pints, 25 are 5 times 5, 10 are twice 5, 5 times 5 are 5 times the half of twice 5; therefore if the cost of 10 pints be 2*s.* 6*d*, the cost of 25 pints, which are 5 times the half of 10, will be 5 times the half of 2*s.* 6*d.* or 6*s.* 3*d.*

iii. If 4 cwt. 3 stones of potatoes cost 14*s.* 7*d.*, what will 15 stones cost ?....*Ans.* 3 times the seventh of 14*s.* 7*d.* or 6*s.* 3*d.*

Proof. 4 cwt. 3 stones are 35 stones; 15 are 3 times 5, 35 are 7 times 5, 3 times 5 are 3 times the seventh of 7 times 5; therefore if the cost of 35 stones be 14*s*. 7*d*., the cost of 15 stones, which are 3 times the seventh of 35 stones, will be 3 times the seventh of 14*s*. 7*d*. or 6*s*. 3*d*.

iv. 3 lbs. 7 oz. of tea cost 14*s*. 8*d*., what will 1 lb. 9 oz. cost ?....*Ans*. 5 times the eleventh of 14*s*. 8*d*. or 6*s*. 8*d*.

Proof. 3 lb. 7 oz. are equal to 55 oz.; 1 lb. 9 oz. are equal to 25 oz.; then 25 are 5 times 5, 55 are 11 times 5, 5 times 5 are 5 times the eleventh of 11 times 5; therefore if 55 oz. cost 14*s*. 8*d*., 25, which are 5 times the eleventh of 55, will cost 5 times the eleventh of 14*s*. 8*d*. or 6*s*. 8*d*.

v. What cost 3 yds. 3 qrs. of cloth at 2*l*. 10*s*. for 2 yds. 2 qrs. ?....*Ans*. 3 times the half of 2*l*. 10*s*. or 3*l*. 15*s*.

Proof. 3 yds. 3 qrs. are equal to 15 qrs., 2 yds. 2 qrs. are equal to 10 qrs., 15 are 3 times 5, 10 are twice 5, 3 times 5 are 3 times the half of twice 5; therefore 15, which are 3 times the half of 10, will cost 3 times the half of 2*l*. 10*s*. or 3*l*. 15*s*.

vi. 10 oz. are what part of 2 lbs. 3 oz. ?....*Ans*. Twice the seventh.

Proof. 2 lbs. 3 oz. contain 35 oz.; 10 are twice 5, 35 are 7 times 5, twice 5 are twice the seventh of 7 times 5.

vii. 15*s*. are what part of a pound ?....*Ans*. 3 times the fourth.

Proof. 15 are 3 times 5, 20 are 4 times 5, 3 times 5 are 3 times the fourth of 4 times 5.

viii. If a man earn 3*l*. in 15 days, what will he earn in 25 days at the same rate ?....*Ans*. 5*l*.

Proof. 25 are 5 times 5, 15 are 3 times 5, 5 times 5 are 5 times the third of 3 times 5; then as 25 days are 5 times the third of 15 days, in 25 days the man will earn 5 times the third of 3*l*., that is, 5*l*.

ix. If 10 masons can build 22 yards of walling in 1 day, how many yards will 25 masons build in the same time? ....*Ans*. 55 yards.

Proof. 25 are 5 times the half of 10; therefore the number of yards built by 25 men will be 5 times the half of 22, that is, 55 yards.

Case III. *When a given number is a given part of a number required.*

Ex. Of what number is 6 three times the fifth ?....*Ans.* 10.

Questions of this kind may be readily solved by a slight variation of the preceding exercise. A few examples will render this apparent. Taking the above example we have "6 are 3 times 2, 3 times 2 are 3 times the fifth of 5 times 2 or 10." Thus the first step consists in finding what number taken 3 times will produce 6; this number must be 2; but the number thus found is the fifth of the required one, and hence the number sought must be 5 times 2 or 10.

In going over this process, the teacher first points to the 6 marks upon the line of twos, which will show of what number 6 is 3 times; with another pointer he shows of what number 2 is the fifth part; so that when the first pointer is placed on the third square, the pupils say "6 are 3 times 2;" when the second pointer is put on the fifth square, then the pupils say, "3 times 2 are 3 times the fifth of 5 times 2, or 10."

The questions belonging to this Case being peculiar in their form, the teacher may find it necessary to break them up in the manner of which the following is an instance :—Let it be required to show of what number 8 are 4 times the third part.

Teacher. (Placing his pointer upon the fourth square of the second line.) In giving the proof of this question, why do I place my pointer upon the fourth square ?

Pupils. Because we have first to find what number of units multiplied by 4 produces 8, which we there find to be 2.

Teacher. Upon what square must I now place my pointer, in order to show of what number these two units are the third ?

Pupils. Upon the third square, which shows that 2 are the third of 6.

Teacher. Then 8 are 4 times ——

Pupils. Two.

Teacher. And 8 are 4 times the third of what number ?

Pupils. Six.

Questions on Case III.

First Line.

i. 3 are 3 times the seventh of what number ?....*Ans.* 7.
Proof. 3 are 3 times 1, 3 times 1 are 3 times the seventh of 7 times 1, or 7.

ii. 5 are 5 times the eighth of what number ?....*Ans.* 8.
Proof. 5 are 5 times 1, 5 times 1 are 5 times the eighth of 8 times 1 or 8.

Second Line.

i. 6 are 3 times the fifth of what number ?....*Ans.* 10.
Proof. 6 are 3 times 2, 3 times 2 are 3 times the fifth of 5 times 2 or 10.

ii. 10 are 5 times the eighth of what number ?....*Ans.* 16.
Proof. 10 are 5 times 2, 5 times 2 are 5 times the eighth of 8 times 2 or 16.

iii. 12 are 6 times the seventh of what number ?....*Ans.* 14.
Proof. 12 are 6 times 2, 6 times 2 are 6 times the seventh of 7 times 2 or 14.

Third Line.

i. 6 are twice the third of what number ?....*Ans.* 9.
Proof. 6 are twice 3, twice 3 are twice the third of 3 times 3 or 9.

ii. 15 are 5 times the ninth of what number ?....*Ans.* 27.
Proof. 15 are 5 times 3, 5 times 3 are 5 times the ninth of 9 times 3 or 27.

iii. 18 are 6 times the fourth of what number ?....*Ans.* 12.
Proof. 18 are 6 times 3, 6 times 3 are 6 times the fourth of 4 times 3 or 12.

Fourth Line.

i. 16 are 4 times the seventh of what number ?....*Ans.* 28.
Proof. 16 are 4 times 4, 4 times 4 are 4 times the seventh of 7 times 4 or 28.

ii. 32 are 8 times the fifth of what number ?....*Ans.* 20.
Proof. 32 are 8 times 4, 8 times 4 are 8 times the fifth of 5 times 4 or 20.

iii. 12 are 3 times the ninth of what number ?....*Ans.* 36.
Proof. 12 are 3 times 4, 3 times 4 are 3 times the ninth of 9 times 4 or 36.

Fifth Line.

i. 10 are twice the fifth of what number?....*Ans.* 25.

Proof. 10 are twice 5, twice 5 are twice the fifth of 5 times 5 or 25.

ii. 15 are 3 times the eighth of what number?....*Ans.* 40.

Proof. 15 are 3 times 5, 3 times 5 are 3 times the eighth of 8 times 5 or 40.

iii. 45 are 9 times the fourth of what number?....*Ans.* 20.

Proof. 45 are 9 times 5, 9 times 5 are 9 times the fourth of 4 times 5 or 20.

Sixth Line.

i. 42 are 7 times the third of what number?....*Ans.* 18.

Proof. 42 are 7 times 6, 7 times 6 are 7 times the third of 3 times 6 or 18.

ii. 54 are 9 times the fifth of what number?....*Ans.* 30.

Proof. 54 are 9 times 6, 9 times 6 are 9 times the fifth of 5 times 6 or 30.

iii. 24 are 4 times the seventh of what number?....*Ans.* 42.

Proof. 24 are 4 times 6, 4 times 6 are 4 times the seventh of 7 times 6 or 42.

Questions on Money, Weights, and Measures.

First Line.

i. 7*d.* are 7 times the third of what sum?....*Ans.* 3*d.*

Proof. 7*d.* are 7 times 1*d.*, 7 times 1*d.* are 7 times the third of 3 times 1*d.* or 3*d.*

ii. 8*s.* are 8 times the fifth of how many shillings?....*Ans.* 5*s.*

Proof. 8*s.* are 8 times 1*s.*, 8 times 1*s.* are 8 times the fifth of 5 times 1*s.* or 5*s.*

iii. If the length of 4 times the fifth of a piece of timber be 4 feet, what is the length of the whole?....*Ans.* 5 feet.

Proof. 4 are 4 times 1, or 4 times the fifth of 5 times 1 or 5.

Second Line.

i. 1 lb. 2 oz. are 9 times the sixth of how many oz.?.... *Ans.* 12 oz.

Proof. In 1 lb. 2 oz. there are 18 oz.; 18 oz. are 9 times 2 oz., 9 times 2 oz. are 9 times the sixth of 6 times 2 oz. or 12 oz.

ii. 8*d.* are 4 times the tenth of what sum?....*Ans.* 1*s.* 8*d.*

Proof. 8*d.* are 4 times 2*d.*, 4 times 2*d.* are 4 times the tenth of 10 times 2*d.* or 20*d.*; 20 pence are 1*s.* 8*d.*

iii. If 3 times the fourth of a lb. of sugar cost 6*d.*, what is the price per lb.?....*Ans.* 8*d.*

Proof. 6 are 3 times 2, 3 times 2 are 3 times the fourth of 4 times 2 or 8. Or thus, if 3 quarters cost 6*d.*, one quarter will cost the third of 6*d.* or 2*d.*; and therefore 4 quarters or 1 lb. will cost 4 times 2*d.* or 8*d.*

Third Line.

i. 1*s.* 9*d.* are 7 times the ninth of what sum?....*Ans.* 2*s.* 3*d.*

Proof. In 1*s.* 9*d.* there are 21 pence, 21 pence are 7 times 3*d.*, 7 times 3*d.* are 7 times the ninth of 9 times 3*d.* or 27 pence, 27 pence are 2*s.* 3*d.*

ii. $2\frac{1}{4}$ yards are 3 times the seventh of how many yards? ....*Ans.* $5\frac{1}{4}$ yards.

Proof. In $2\frac{1}{4}$ yards there are 9 quarters, 9 quarters are 3 times 3 quarters, 3 times 3 quarters are 3 times the seventh of 7 times 3 quarters or 21 quarters, and 21 quarters are 5 yards and 1 quarter.

iii. A post is 6 feet in the earth, and this part is twice the fifth of the whole length; required the height above the ground.....*Ans.* 9 feet.

Proof. 6 are twice 3, twice 3 are twice the fifth of 5 times 3 or 15; therefore 15 feet being the whole length, the height above the ground is $15 - 6 = 9$ feet.

Fourth Line.

i. 1*s.* 4*d.* are 4 times the tenth of what sum?....*Ans.* 3*s.* 4*d.*

Proof. In 1*s.* 4*d.* there are 16 pence, 16*d.* are 4 times 4*d.*, 4 times 4*d.* are 4 times the tenth of 10 times 4*d.* or 40 pence. 40*d.* are 3*s.* 4*d.*

ii. 2 lb. 4 oz. are 9 times the sixth of what quantity? ....*Ans.* 1 lb. 8 oz.

Proof. 2 lb. 4 oz. are 36 oz., 36 oz. are 9 times 4 oz., 9 times 4 oz. are 9 times the sixth of 6 times 4 oz. or 24 oz., 24 oz. = 1 lb. 8 oz.

Fifth Line.

i. 2*s*. 11*d*. are 7 times the ninth of what sum?....*Ans.* 3*s*. 9*d*.

Proof. In 2*s*. 11*d*. there are 35 pence, 35 are 7 times 5, 7 times 5 are 7 times the ninth of 9 times 5 or 45; 45*d*. are 3*s*. 9*d*.

ii. 1*l*. 10*s*. is 6 times the fifth of what sum?....*Ans.* 1*l*. 5*s*.

Proof. In 1*l*. 10*s*. there are 30*s*., 30*s*. are 6 times 5*s*., 6 times 5*s*. are 6 times the fifth of 5 times 5 or 25*s*., 25*s*. are 1*l*. 5*s*.

Sixth Line.

i. 4 yards and 2 quarters are 3 times the seventh of how many yards?....*Ans.* 10 yards 2 qrs.

Proof. In 4 yards 2 quarters there are 18 quarters, 18 quarters are 3 times 6 quarters, 3 times 6 quarters are 3 times the seventh of 7 times 6 quarters or 42 quarters, 42 quarters are 10 yards and 2 quarters.

ii. 1*l*. 16*s*. are 6 times the tenth of what sum?....*Ans.* 3*l*.

Proof. In 1*l*. 16*s*. there are 36*s*., 36*s*. are 6 times 6*s*., 6 times 6*s*. are 6 times the tenth of 10 times 6*s*. or 60*s*., 60*s*. are 3*l*.

Miscellaneous Questions on the Fifth Exercise.

The answers to the following questions contain the proof of many of the common rules of mental arithmetic.

i. How many dozens are contained in 30?....*Ans.* 2 dozens and the half of a dozen.

Proof. 30 are twice 12 and the half of 12.

ii. 32*s*. contain how many pounds?....*Ans.* One pound and 3 times the fifth of a pound.

Proof. 32 contain 20, once and 12 remaining, 12 are 3 times the fifth of 20, therefore 32 are once 20 and 3 times the fifth of 20.

iii. 65*d.* are how many shillings?....*Ans.* 5*s.* and 5 times the twelfth of a shilling.

iv. How many cwt. are there in 10 qrs.?....*Ans.* 2 cwt. and a half.

Proof. 10 are twice 4 and the half of 4.

v. How many dozens are there in 39?....*Ans.* 3 dozens and a fourth.

Proof. 39 are 3 times 12 and the fourth of 12.

vi. If 4 men can build a wall in 5 days, how long would 2 men be in building it?....*Ans.* 10 days.

Proof. 2 is the half of 4, therefore 2 men will take twice the time which 4 men take; and twice 5 are 10.

vii. How long would 10 men require to perform the work in the last example?....*Ans.* 2 days.

Proof. 4×5 give 20 days' work; 10 men do 10 days' work in one day, and therefore they require 2 days to complete 20 days' work.

viii. What is the price of 12 articles at 8*d.* each?.... *Ans.* 8*s.*

Proof. 12 at 1*d.* each are 12*d.* or 1*s.*, and 12 at 8*d.* are 8 times 12*d.* or 8*s.*

ix. What is the price of 24 articles at 1*s.* 2*d.* each?....*Ans.* 1*l.* 8*s.*

Proof. 1*s.* 2*d.*=14*d.*; 24 at 1*d.* are 2*s.*, and at 14*d.* each, are 14 times 2*s.*, or 1*l.* 8*s.*

x. What is the price of 36 articles at 2*s.* 4*d.* each?.... *Ans.* 4*l.* 4*s.*

Proof. 2*s.* 4*d.* are 28*d.*; the cost of 12 articles at 1*d.* each is 1*s.*, therefore the cost of 12 at 28*d.* each, will be 28*s.*, and the cost of 36, which is 3 times 12, will be 3 times 28*s.*, or 4*l.* 4*s.*

xi. Find the price of 48 articles at 1*s.* 9*d.* each. ....*Ans.* 4*l.* 4*s.*

xii. What is the price of 24 lbs. at 2*s.* 8*d.* per lb.?....*Ans.* 3*l.* 4*s.*

xiii. What is the price of 60 articles at 5*d.* each?....*Ans.* 1*l.* 5*s.*

xiv. Find the price of 72 oz. at 4*d.* per oz.....*Ans.* 1*l.* 4*s.*

xv. What is the amount of 42 lbs. of sugar at 8*d.* per lb.? ....*Ans.* 1*l.* 8*s.*

Proof. 1 dozen at 8*d.* each, amounts to 8*s.*, and 42, which is $3\frac{1}{2}$ dozen, will be 3 times 8*s.* and $\frac{1}{2}$ of 8*s.*, or 28*s.* = 1*l.* 8*s.*

xvi. At 1*s*. 4*d*. per lb., what cost 38 lbs.?....*Ans*. 2*l*. 10*s*. 8*d*.

Proof. 1*s*. 4*d*. are 16*d*.; 38 are 3 dozen and 2; 3 dozen amount to 3×16, or 48*s*.; and 2 lbs. cost twice 1*s*. 4*d*., or 2*s*. 8*d*.; 48*s*. and 2*s*. 8*d*. make 2*l*. 10*s*. 8*d*.

xvii. $72\frac{1}{2}$ at 8*d*. ....*Ans*. 2*l*. 8*s*. 4*d*.

Proof. 72 are 6 dozens. The cost of 1 dozen at 8*d*. is 8*s*., therefore the cost of 6 dozens will be 6 times the cost of 1, or 6 times 8*s*.=48*s*. If one dozen cost 8*d*., one half dozen will cost 4*d*. The amount of $72\frac{1}{2}$ will therefore be 48*s*. 4*d*., or 2*l*. 8*s*. 4*d*.

xviii. What is the amount of $48\frac{3}{4}$ lbs. of coffee at 2*s*. 4*d*. per lb.?....*Ans*. 5*l*. 13*s*. 9*d*.

Proof. The cost of 48, or 4 dozen, is 5*l*. 12*s*.; and 3 times the fourth of 2*s*. 4*d*. are 3 times 7*d*., or 21*d*. = 1*s*. 9*d*., which added to 5*l*. 12*s*., gives 5*l*. 13*s*. 9*d*.

xix. At 3*d*. per lb., how much per cwt.?....*Ans*. 1*l*. 8*s*.

Proof. At 1*d*. per lb., a cwt. will cost 112*d*., or 9*s*. 4*d*.; and therefore 3*d*. per lb. will amount to 3 times 9*s*. 4*d*., or 1*l*. 8*s*.

xx. At $3\frac{3}{4}$*d*. per lb., how much per cwt.?....*Ans*. 1*l*. 15*s*.

Proof. At 3*d*. per lb., a cwt. amounts to 1*l*. 8*s*.; and as 1*d*. per lb. amounts to 112*d*. per cwt., the cost of a cwt. at $\frac{3}{4}$*d*. per lb. will be 3 times the fourth of 112*d*., or 7*s*.; and 7*s*. added to 1*l*. 8*s*. are 1*l*. 15*s*.

xxi. At 4*d*. per lb., what is the cost of a ton?....*Ans*. 37*l*. 6*s*. 8*d*.

Proof. 4*d*. per lb. will be 4 times 9*s*. 4*d*. per cwt.= 37*s*. 4*d*.; a ton at 1*s*. per cwt. will amount to 20*s*., or 1*l*.; and a ton at 37*s*. per cwt will come to 37 times 1*l*., or 37*l*.; 4*d*. is the third part of 1*s*., therefore 4*d*. per cwt. will be the third of 1*l*., or 6*s*. 8*d*. per ton; then 37*l*. and 6*s*. 8*d*. make 37*l*. 6*s*. 8*d*.

xxii. What part of a cwt. is 2 qrs.?....*Ans*. The half.

Proof. On the second line.

xxiii. What part of a lb. is 12 oz.?....*Ans*. 3 times the fourth.

Proof. On the line of fours.

xxiv. What part of a shilling is 8*d*.?....*Ans*. Twice the third.

Proof. On the line of fours.

xxv. What part of a pound is 13*s*. 4*d*.?....*Ans*. Twice the third.

xxvi. What is the interest of 120*l.* at 6 per cent.?....*Ans.* 7*l.* 4*s.*

Proof. 120 are 6 times the fifth of 100; the interest upon 100 is 6*l.*, therefore the interest upon 120 will be 6 times the fifth of 6*l.*; the fifth of 6*l.* is 1*l.* 4*s.*, and 6 times the fifth of 6*l.* will be 6 times 1*l.* 4*s.*, or 7*l.* 4*s.*

xxvii. If the interest upon 100*l.* be 5*l.*, what is the interest upon 1*l.*, 10*s.*, 5*s.*, 2*s.* 6*d.*, 1*s.* 3*d.*, and 5*d.*?....*Ans.* 1*s.*, 6*d.*, 3*d.*, $1\frac{1}{2}d.$, $\frac{3}{4}d.$, and $\frac{1}{4}d.$

Proof. 5*l.* contain 100 shillings; as the interest upon 100*l.* is 100 shillings, the interest of 1*l.* will be 1 shilling. In like manner, the interest upon 10*s.*, which is the half of 1*l.*, will be the half of 1*s.*, or 6*d.*; upon 5*s.* will be 3*d.*; upon 2*s.* 6*d.* will be $1\frac{1}{2}d.$; upon 1*s.* 3*d.* will be $\frac{3}{4}d.$; and upon 5*d.* will be $\frac{1}{4}d.$

xxviii. If goods are bought for 15*l.*, and sold for 21*l.*; what is the gain per cent.?....*Ans.* 40 per cent.

Proof. On the line of fives. The gain upon 15*l.* is 6*l.*; but 6 are twice the 5th of 15, therefore the gain upon 100*l.* will be twice the fifth of 100, or 40*l.*

xxix. If the gain upon 1 lb. of tea be 8*d.*, what is the gain upon 12 oz.?....*Ans.* 6*d.*

Proof. On the line of fours.

xxx. Of what number of shillings and pence are 6*s.* 3*d.* 3 times the fourth part?....*Ans.* 8*s.* 4*d.*

Proof. 6*s.* 3*d.* are 3 times 2*s.* 1*d.*, or 3 times the fourth of 8*s.* 4*d.*

xxxi. If 9 men working for 5 days earn 4*l.* 10*s.*, how much would 6 men earn working for 15 days?....*Ans.* 9*l.*

Proof. 9 men, working for 5 days, will do 45 days' work; 6 men, working for 15 days, will do 90 days' work; 90 are twice 45; therefore the 6 men, in 15 days, will earn twice 4*l.* 10*s.*, that is, 9*l.*

xxxii. A yard of cloth 4 quarters wide is worth 12*s.*; what is the value of a yard 5 quarters wide?....*Ans.* 15*s.*

Proof. 5 are 5 times the fourth of 4; therefore the value of the cloth which is 5 quarters wide will be 5 times the fourth of 12*s.*, which is 15*s.*

xxxiii. An estate is worth 7000*l.*, what part of it can be purchased for 2000?....*Ans.* Twice the seventh.

Proof. 2000 are twice the seventh of 7000.

xxxiv. A grocer mixes 10 lbs. of tea at 5*s*. with 5 lbs. at 8*s*.; required the worth of the mixture per lb. ....*Ans*. 6*s*.

Proof. The cost of the mixture will be 90*s*.; then if 15 lbs. cost 90*s*., 1 lb. will cost the fifteenth part of 90*s*., that is, 6*s*.

xxxv. If a man earn 30*s*. per week, what wages will he receive for 5 days' work?....*Ans*. 1*l*. 5*s*.

Proof. 5 are 5 times the sixth of 6, therefore the wages for 5 days will be 5 times the sixth of 30*s*., that is, 1*l*. 5*s*.

xxxvi. If a labourer can lift 500 feet of earth in one day, what time will he require to lift 1500 feet?....*Ans*. 3 days.

Proof. 1500 are 3 times 500; if, therefore, a man take 1 day in lifting 500 feet, he will take 3 times 1 day, or 3 days, in lifting 1500 feet.

xxxvii. An engine raises 3000 feet of water in an hour; how much will be raised in 1 minute?....*Ans*. 50 feet.

Proof. 1 minute is the sixtieth part of an hour; in 1 minute, therefore, will be raised the sixtieth part of 3000 feet, that is, 50 feet.

SIXTH EXERCISE.

Proportion.

The nature of ratios, as explained in the last exercise, having been thoroughly understood by the pupil, he will now be prepared to consider the subject of Proportion. The present exercise is intended to show that, when the ratio of two numbers is the same as that of two other numbers, then these four numbers form a proportion: thus, as the ratio of 7 to 28 is the same as 4 to 16, we have the proportion,

7 is to 28 as 4 is to 16;

or, 7 is to 4 times 7 or 28 as 4 is to 4 times 4 or 16.

Again, 4 is to 6 as 8 is to 12;

or, 4 is to 3 times the half of 4 or 6 as 8 is to 3 times the half of 8 or 12.

In general, the pupil will be taught, that if the first term is contained the same number of times, or parts of times, in the second, that the third is contained in the fourth, then these four numbers form a proportion.

The application of this principle is highly important: for when the first three numbers, or terms, of a proportion are given, the pupil is enabled to find the fourth number or term. For example, in the question, 8 is to 12 as 10 is to what number? the number required will have the same ratio to 10 that 12 has to 8. The pupil, therefore, first finds the ratio of 12 to 8, which is 3 times the half, and then goes on to state,

"8 is to 3 times the half of 8 or 12 as 10 is to 3 times the half of 10 or 15."

Hence he finds the number required to be 15.

The teacher will not fail to observe, that while the first and third terms of the proportion are taken in the same column upon the board, the last, or required term, will always be found in the same column with the second term, a circumstance which imparts a beautiful simplicity to the subject of proportion.

For the sake of convenience in teaching, the forms of proportion may be divided into three kinds:—1st. When the second term is divisible by the first; 2nd. When the first term is divisible by the second; 3d. When the first and second terms have any ratio.

Case I. *When the second term is divisible by the first.*

The manner in which the teacher is to proceed throughout the following exercises may be illustrated by an example. Suppose that he is about to show that "2 is to 5 times the half of 2 or 5, as 4 is to 5 times the half of 4 or 10."

While the teacher puts one pointer upon the second, and the other upon the fifth square of the first line, the pupils say "5 are 5 times the half of 2,—2 is to 5 times the half of 2 or 5;" then the teacher points to the two squares immediately below, and the pupils say, "as 4 is to 5 times the half of 4 or 10."

In the course of the exercise such questions as the following may be put:

Ques. What is the truth which you have just established? ....*Ans.* That 2 is to 5 as 4 is to 10.

Ques. What constitutes these numbers in proportion?.... *Ans.* That 5 contains 2 the same number of times that 10 contains 4.

Ques. How many times 2 is five ?....*Ans.* 5 times the half.

Ques. How many times 4 is ten ?....*Ans.* 5 times the half also; because the half of 4, which is 2, taken 5 times produces 10.

Ques. Give me any four numbers in proportion.

First and Second Columns.

1 is to twice 1 or 2 as 2 is to twice 2 or 4
2 : twice 2 or 4 :: 3 : twice 3 or 6
3 : twice 3 or 6 :: 4 : twice 4 or 8
4 : twice 4 or 8 :: 5 : twice 5 or 10
And so on to
9 : twice 9 or 18 :: 10 : twice 10 or 20

The Columns taken promiscuously.

8 : twice 8 or 16 :: 6 : twice 6 or 12
5 : twice 5 or 10 :: 9 : twice 9 or 18

First and Third Columns.

1 is to 3 times 1 or 3 as 2 is to 3 times 2 or 6
2 : 3 times 2 or 6 :: 3 : 3 times 3 or 9
3 : 3 times 3 or 9 :: 4 : 3 times 4 or 12
4 : 3 times 4 or 12 :: 5 : 3 times 5 or 15
And so on to
9 : 3 times 9 or 27 :: 10 : 3 times 10 or 30

The Columns taken promiscuously.

7 : 3 times 7 or 21 :: 10 : 3 times 10 or 30
4 : 3 times 4 or 12 :: 9 : 3 times 9 or 27

First and Fourth Columns.

1 is to 4 times 1 or 4 as 2 is to 4 times 2 or 8
2 : 4 times 2 or 8 :: 3 : 4 times 3 or 12
3 : 4 times 3 or 12 :: 4 : 4 times 4 or 16
4 : 4 times 4 or 16 :: 5 : 4 times 5 or 20
And so on to
9 : 4 times 9 or 36 :: 10 : 4 times 10 or 40

The Columns taken promiscuously.

7 : 4 times 7 or 28 :: 3 : 4 times 3 or 12
5 : 4 times 5 or 20 :: 9 : 4 times 9 or 36
2 : 4 times 2 or 8 :: 6 : 4 times 6 or 24

In like manner the first and fourth, first and fifth, first and sixth, first and seventh, first and eighth, first and ninth, and first and tenth columns may be gone over.

Questions on Case I.

First and Second Columns.

i. 3 : 6 :: 8 : what number?....*Ans.* 16.

Proof. 6 are twice 3; then 3 : twice 3 or 6 :: 8 : twice 8, or 16.

ii. 4 : 8 :: 12 : an unknown number?....*Ans.* 24.

Proof. 8 are twice 4; hence 4 : twice 4 or 8 :: 12 : twice 12, or 24.

iii. 5 : 10 :: 9 : what number?....*Ans.* 18.

iv. 6 : 12 :: 5 : what number?....*Ans.* 10.

v. 10 : 20 :: 3 : what number?....*Ans.* 6.

First and Third Columns:

i. 2 : 6 :: 4 : what number?....*Ans.* 12.

Proof. 6 are three times 2; hence 2 · 3 times 2, or 6 :: 4 : 3 times 4, or 12.

ii. 5 : 15 :: 6 : what number?....*Ans.* 18.

iii. 4 : 12 :: 7 : what number?....*Ans.* 21.

iv. 10 : 30 :: 8 : what number?....*Ans.* 24.

First and Fourth Columns.

i. 3 : 12 :: 5 : what number?....*Ans.* 20.

Proof. 12 are 4 times 3; therefore, 3 : 4 times 3, or 12 :: 5 : 4 times 5, or 20.

ii. 5 : 20 :: 2 : what number?....*Ans.* 8.

iii. 7 : 28 :: 3 : what number?....*Ans.* 12.

iv. 9 : 36 :: 10 : what number?....*Ans.* 40.

Similar questions may be given on the first and fifth, first and sixth, &c., columns.

Questions on Money, Weights, and Measures.

First and Second Columns.

i. If 6 articles cost 2*s*. 8*d*., what will 12 cost?....*Ans*. 5*s*. 4*d*.

Proof. 2*s*. 8*d*. are 32*d*.; 12 are twice 6; therefore, 6 : twice 6, or 12 :: 32 : twice 32, or 64; 64*d*. are 5*s*. 4*d*.

ii. If 8 cost 2*s*. 5*d*., what will 16 cost?....*Ans*. 4*s*. 10*d*.

iii. If 5 cost 1*s*. 8*d*., what will 10 cost?....*Ans*. 3*s*. 4*d*.

iv. If 9 cost 10*d*.; what will 18 cost?....*Ans*. 1*s*. 8*d*.

v. If 4 cost 2*d*., what will 8 cost?....*Ans*. 4*d*.

First and Third Columns.

i. If 2lbs. of tea cost 9*s*., what will 6lbs. come to?....*Ans*. 1*l*. 7*s*.

Proof. 6 are 3 times 2; then 2 : 3 times 2 or 6 :: 9 : 3 times 9, or 27; 27 shillings are 1*l*. 7*s*.

ii. If 2 cost 3*s*. 6*d*., what will 6 cost?....*Ans*. 10*s*. 6*d*.

iii. If 9 cost 2*s*. 1*d*., what will 27 cost?....*Ans*. 6*s*. 1*d*.

iv. If 5 cost 10*d*., what will 15 cost?....2*s*. 6*d*.

First and Fourth Columns.

i. Required the value of 12 yards of cloth, when 3 yards cost 8*s*.....*Ans*. 1*l*. 12*s*.

Proof. 12 are 4 times 3, therefore, 3 : 4 times 3, or 12 :: 8 : 4 times 8, or 32; 32 shillings are 1*l*. 12*s*.

ii. If 2 cost 2*s*. 2¼*d*., what will 8 cost?....*Ans*. 8*s*. 9*d*.

iii. If 3 cost 10*d*., what will 12 cost?....*Ans*. 3*s*. 10*d*.

iv. If 7 cost 2*s*. 2½*d*., what will 28 cost?....*Ans*. 8*s*. 10*d*.

And so on to the first and fifth, first and sixth, &c., columns.

Case II. *When the first term is divisible by the second.*

Second and First Columns.

4 is to the half of 4 or 2 as 2 is to the half of 2 or 1
6 : the half of 6 or 3 :: 4 : the half of 4 or 2
8 : the half of 8 or 4 :: 6 : the half of 6 or 3
10 : the half of 10 or 5 :: 8 : the half of 8 or 4
And so on to
20 : the half of 20 or 10 :: 18 : the half of 18 or 9

The Columns taken promiscuously.

12 : the half of 12 or 6 :: 16 : the half of 16 or 8
18 : the half of 18 or 9 :: 10 : the half of 10 or 5

Third and First Columns.

6 is to the third of 6 or 2 as 3 is to the third of 3 or 1
9 : the third of 9 or 3 :: 6 : the third of 6 or 2
12 : the third of 12 or 4 :: 9 : the third of 9 or 3
15 : the third of 15 or 5 :: 12 : the third of 12 or 4
And so on to
30 : the third of 30 or 10 :: 27 : the third of 27 or 9

The Columns taken promiscuously.

30 : the third of 30 or 10 :: 21 : the third of 21 or 7
27 : the third of 27 or 9 :: 12 : the third of 12 or 4

These illustrations will be sufficient to show how the remaining columns are to be treated.

Questions on Case II.

Second and First Columns.

i. 14 : 7 :: 10 : what number?....*Ans.* 5.

Proof. 7 are the half of 14; therefore 14 : the half of 14, or 7 :: 10 : the half of 10, or 5.

ii. 10 : 5 :: 8 : what number?....*Ans.* 4.

iii. 4 : 2 :: 8 : what number?....*Ans.* 4.

iv. 6 : 3 :: 2 : what number?....*Ans.* 1.

v. 8 : 4 :: 4 : what number?....*Ans.* 2.

Third and First Columns.

i. 9 : 3 :: 6 : what number?....*Ans.* 2.

Proof. 3 are the third of 9; therefore 9 : the third of 9, or 3 :: 6 : the third of 6, or 2.

ii. 3 : 1 :: 12 : what number?....*Ans.* 4.

iii. 27 : 9 :: 15 : what number?....*Ans.* 5.

iv. 21 : 7 :: 30 : what number?....*Ans.* 10.

v. 18 : 6 :: 6 : what number?....*Ans.* 2.

Fourth and First Columns.

i. 8 : 2 :: 12 : what number?....*Ans.* 3.

Proof. 2 are the fourth of 8; therefore, 8 : the fourth of 8, or 2 :: 12 : the fourth of 12 or 3.

ii. 28 : 7 :: 16 : what number?....*Ans.* 4.

iii. 24 : 6 :: 32 : what number?....*Ans.* 8.

iv. 8 : 2 :: 36 : what number?....*Ans.* 9.

v. 20 : 5 :: 4 : what number?....*Ans.* 1.

Similar questions may be given on the first and fifth, first and sixth, &c., columns.

Questions on Money, Weights, and Measures.

Second and First Columns.

i. At 14*s*. per week, how much will 3 days' work amount to?....*Ans.* 7*s*.

Proof. 3 are the half of 6; therefore, 6 : the half of 6 or 3 :: 14 : the half of 14 or 7.

ii. If 14 cost 4*s*. 2*d*., what will 7 cost?....*Ans.* 2*s*. 1*d*.

iii. If 14 cost 10*d*., what will 7 cost?....*Ans.* 5*d*.

iv. If 10 cost 4*s*. 6*d*., what will 5 cost?....*Ans.* 2*s*. 3*d*.

Third and First Columns.

i. If 9 cost 3*s*. 3*d*., what will 3 cost?....*Ans.* 1*s*. 1*d*.

Proof. 3 are the third of 9; therefore, 9 : the third of 9 or 3 :: 3*s*. 3*d*. : the third of 3*s*. 3*d*., or 1*s*. 1*d*.

ii. If 15 cost 2*s*. 9*d*., what will 5 cost?....*Ans.* 11*d*.

iii. If 12 cost 1*s*. 9*d*., what will 4 cost?....*Ans.* 7*d*.

iv. If 6 cost 3*s*. 9*d*., what will 2 cost?....*Ans.* 1*s*. 3*d*.

Fourth and First Columns.

i. The gain upon the sale of 16 lbs. is 2*s*. 8*d*., what would be the gain upon 4 lbs.?....*Ans.* 8*d*.

Proof. 2*s*. 8*d*. are 32*d*.; 4 is the fourth of 16; then 16 : the fourth of 16, or 4 :: 32 : the fourth of 32, or 8.

ii. If 12 cost 2*s*. 4*d*., what will 3 cost?....*Ans.* 7*d*.

iii. If 24 cost 4*s*. 4*d*., what will 6 cost?....*Ans.* 1*s*. 1*d*.

iv. If 28 cost 1*s*. 8*d*., what will 7 cost?....*Ans.* 5*d*.

And so on to the first and fifth, first and sixth, &c., columns.

Case III. *When the first and second terms have any ratio.*

Second and Third Columns.

3 are 3 times the half of 2*
2 : 3 times the half of 2, or 3 :: 4 : 3 times the half of 4 or 6
6 are 3 times the half of 4*
4 : 3 times the half of 4, or 6 :: 6 : 3 times the half of 6 or 8
9 are 3 times the half of 6*
6 : 3 times the half of 6, or 9 :: 8 : 3 times the half of 8 or 12
And so on.

Second and Fourth Columns.

4 are 4 times the half of 2
2 : 4 times the half of 2, or 4 :: 4 : 4 times the half of 4 or 8
8 are 4 times the half of 4
4 : 4 times the half of 4, or 8 :: 6 : 4 times the half of 6 or 12
12 are 4 times the half of 6
6 : 4 times the half of 6, or 12 :: 8 : 4 times the half of 8 or 16.
And so on.

Second and Fifth Columns.

5 are 5 times the half of 2
2 : 5 times the half of 2, or 5 :: 4 : 5 times the half of 4 or 10
10 are 5 times the half of 4
4 : 5 times the half of 4, or 10 :: 6 : 5 times the half of 6, or 15
15 are 5 times the half of 6
6 : 5 times the half of 6, or 15 :: 8 : 5 times the half of 8, or 20
And so on.

In like manner the second and sixth, second and seventh, &c., may be gone over.

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* By omitting the decomposition placed at the commencement of each step, the connection between the different steps will be more easily seen.

Third and Second Columns.

2 are twice the third of 3
3 : twice the third of 3, or 2 :: 6 : twice the third of 6, or 4
4 are twice the third of 6
6 : twice the third of 6, or 4 :: 9 : twice the third of 9, or 6
6 are twice the third of 9
9 : twice the third of 9, or 6 :: 12 : twice the third of 12, or 8
And so on.

Third and Fourth Columns.

4 are 4 times the third of 3
3 : 4 times the third of 3, or 4 :: 6 : 4 times the third of 6, or 8
8 are 4 times the third of 6
6 : 4 times the third of 6, or 8 :: 9 : 4 times the third of 9, or 12
12 are 4 times the third of 9
9 : 4 times the third of 9, or 12 :: 12 : 4 times the third of 12, or 16
And so on.

In like manner the third and fifth, third and sixth, &c., columns may be gone over, and the same form may be extended to any other combination of the columns. For another example take the

Seventh and Ninth Columns.

9 are 9 times the seventh of 7
7 : 9 times the seventh of 7, or 9 :: 14 : 9 times the seventh of 14, or 18
18 are 9 times the seventh of 14
14 : 9 times the seventh of 14, or 18 :: 21 : 9 times the seventh of 21, or 27
27 are 9 times the seventh of 21
21 : 9 times the seventh of 21, or 27 :: 28 : 9 times the seventh of 28, or 36
36 are 9 times the seventh of 28
28 : 9 times the seventh of 28, or 36 :: 35 : 9 times the seventh of 35, or 45
And so on.

The following form may also be used with advantage:

Ninth and Seventh Columns.

9 are 9 times the seventh of 7
18 are 9 times the seventh of 14
18 : 14 :: 9 : 7

9 are 9 times the seventh of 7
27 are 9 times the seventh of 21
27 : 21 :: 9 : 7

9 are 9 times the seventh of 7
36 are 9 times the seventh of 28
36 : 28 :: 9 : 7

9 are 9 times the seventh of 7
45 are 9 times the seventh of 35
45 : 35 :: 9 : 7

And so on.

Any other two columns may be gone over in the same manner.

Since a proportion is formed by four terms, having an equality of ratio, it follows that we may divide the first and second, or the third and fourth terms by any number without altering the proportion.

So also in any proportion the product of the middle terms is equal to the product of the extreme terms. Thus in the proportion 9 : 12 :: 6 : 8; we find that $9 \times 8 = 12 \times 6$.

This latter property is the foundation of the common process of "Rule of Three," where three terms of the proportion are given to find the fourth, as in the following example:

2 : 3 :: 8 : a number unknown.

Hence the unknown number $\times 2 = 8 \times 3$, or the unknown number $= \frac{8 \times 3}{2} = 12$.

In general the last, or unknown term of a proportion is found by dividing the product of the middle terms by the first. Having given the value, &c. of any number of things of the same nature, proportion enables us to find the value, &c. of any other number of things of the same kind, and it is evident that two quantities cannot be put into a proportion until it is ascertained that the value, &c. of the quantities considered, change uniformly with the change of the number.

The teacher will find little difficulty in illustrating these properties upon the Board of Simple Units.

Questions on Case III.

Third and Fifth Columns.

i. 6 : 10 :: 9 : what number?....*Ans.* 15.

Proof. 10 are 5 times the third of 6; therefore, by the equality of ratios, 6 : 5 times the third of 6 :: 9 : 5 times the third of 9, or 15.

ii. 21 : 35 :: 15 : an unknown number?....*Ans.* 25.

Proof. 35 are 5 times the third of 21; 21 : 5 times the third of 21, or 35 :: 15 : 5 times the third of 15, or 25.

iii. 18 : 30 :: 6 : what number?....*Ans.* 10.

iv. 25 : 15 :: 5 : an unknown number?....*Ans.* 3.

Fourth and Seventh Columns.

i. 16 : 28 :: 12 : what number?....*Ans.* 21.

Proof. 28 are 7 times the fourth of 16; 16 : 7 times the fourth of 16 :: 12 : 7 times the fourth of 12, or 21.

ii. 35 : 20 :: 21 : an unknown number?....*Ans.* 12.

iii. 14 : 8 :: 42 : what number?....*Ans.* 24.

iv. What proportion does 9 bear to 15?....*Ans.* 3 to 5.

Proof. 15 are 5 times the third of 9; therefore, 9 : 5 times the third of 9 or 15 :: 3 : 5 times the third of 3, or 5.

v. Find the proportion of 12 to 20.....*Ans.* 3 to 5.

Proof. 20 are 5 times the third of 12, therefore 12 : 5 times the third of 12, or 20 :: 3 : 5 times the third of 3, or 5.

Ninth and Seventh Columns.

i. What proportion does 18 bear to 14?....*Ans.* 9 to 7.

ii. What proportion does 27 bear to 21?....*Ans.* 9 to 7.

iii. What proportion does 45 bear to 35?....*Ans.* 9 to 7.

Third and Tenth Columns.

i. What proportion does 6 bear to 20?....*Ans.* 3 to 10.

ii. What proportion does 9 bear to 30?....*Ans.* 3 to 10.

iii. What proportion does 15 bear to 50?....*Ans.* 3 to 10.

iv. What proportion does 21 bear to 70?....*Ans.* 3 to 10.

Questions on Money, Weights, and Measures.

Second and Third Columns.

i. What will be the value of 4 yards of cloth, when 6 yards cost 21*s*. ?....*Ans*. 14*s*.

Proof. 4 are twice the third of 6; then 6 : twice the third of 6, or 4 :: 21 : twice the third of 21, or 14.

ii. If 6 cost 15*s*. 3*d*., what will 4 cost ?....*Ans*. 10*s*. 2*d*.

iii. If 10 cost 8*s*. 0*d*., what will 15 cost ?....*Ans*. 12*s*.

iv. If 14 cost 3*s*. 2*d*., what will 21 cost ?....*Ans*. 4*s*. 9*d*.

v. If 3 cost 1*s*. 6*d*., what will 2 cost ?....*Ans*. 1*s*.

Third and Fourth Columns.

i. At 12 shillings per week, how much will 8 days' work amount to ?....*Ans*. 16*s*.

Proof. 8 are 4 times the third of 6; then 6 : 4 times the third of 6, or 8 :: 12 : 4 times the third of 12, or 16.

ii. If 15 cost 1*s*. 3*d*., what will 20 cost ?....*Ans*. 1*s*. 8*d*.

iii. If 18 cost 8*s*. 9*d*., what will 24 cost ?....*Ans*. 11*s*. 8*d*.

iv. If 9 cost 1*s*. 9*d*., what will 12 cost ?....*Ans*. 2*s*. 4*d*.

v. If 16 cost 2*s*. 0*d*., what will 12 cost ?....*Ans*. 1*s*. 6*d*.

Third and Fifth Columns.

i. If a man walk at the rate of 3 miles per hour, how long would he be in walking 5 miles ?....*Ans*. 1 hour and 40 minutes.

Proof. 5 are 5 times the third of 3; then 3 : 5 times the third of 3 :: 1 : 5 times the third of 1. In one hour there are 60 minutes; therefore 5 times the third of an hour = 100 minutes, or 1 hour and 40 minutes.

ii. If 21 cost 1*s*. 3*d*., what will 35 cost ?....*Ans*. 2*s*. 1*d*.

iii. If 30 cost 3*s*. 6*d*., what will 40 cost ?....*Ans*. 4*s*. 8*d*.

iv. If 12 cost 3*d*., what will 20 cost ?....*Ans*. 5*d*.

Third and Seventh Columns.

i. 6 lbs. of sugar at 7*s.* 7*d.* per stone ?....*Ans.* 3*s.* 3*d.*

Proof. 6 are 3 times the seventh of 14, therefore 14 : 3 times the seventh of 14, or 6 :: 7*s.* 7*d.* : 3 times the seventh of 7*s.* 7*d.*, or 3*s.* 3*d.*

ii. Required the amount of 21 articles, when 9 cost 21*s.* ....*Ans.* 2*l.* 9*s.*

Proof. 21 are 7 times the third of 9; therefore 9 : 7 times the third of 9, or 21 :: 21 : 7 times the third of 21, or 49; 49 shillings are 2*l.* 9*s.*

Similar questions may be given, on any other combination of columns.

Miscellaneous Questions on the Sixth Exercise.

i. If 6 yards of cloth cost 3*l.* 6*s.*, what will 20 yards cost? ....*Ans.* 11*l.*

Proof. Third and tenth columns. 20 yards are 10 times the third of 6 yards, and 3*l.* 6*s.* are 66 shillings; therefore, 6: 10 times the third of 6, or 20 :: 66 : 10 times the third of 66, or 220; 220 shillings are 11*l.*

ii. If 7 yards cost 1*s.* 9*d.*, what would 11 yards cost? ....*Ans.* 2*s.* 9*d.*

Proof. 1*s.* 9*d.* are 21 pence. Hence 7 : 11 times the seventh of 7, or 11 :: 21 : 11 times the seventh of 21, or 33; 33*d.* are 2*s.* 9*d.*

iii. If 64 articles cost 3*s.* 4*d.*, what would 24 cost at the same rate?....*Ans.* 1*s.* 3*d.*

Proof. 24 are 3 times the eighth of 64; 64 : 3 times the eighth of 64, or 24 :: 40 : 3 times the eighth of 40, or 15; and 15*d.* are 1*s.* 3*d.*

iv. If 48 lbs. cost 4*s.* 8*d.*, what would 54 lbs. cost?.... *Ans.* 5*s.* 3*d.*

v. If 72 lbs. cost 3*s.* 4*d.*, what would 54 lbs. cost?.... *Ans.* 2*s.* 6*d.*

vi. If 81 lbs. cost 2*s.* 3*d.*, what would 90 lbs. cost?.... *Ans.* 2*s.* 6*d.*

vii. If 25 lbs. cost 3*s*. 9*d*., what would 40 lbs. cost ?.... *Ans*. 6*s*.

viii. If 112 lbs. cost 1*s*. 9*d*., what would 32 lbs. cost ?.... *Ans*. 6*d*.

ix. If 49 lbs. cost 1*s*. 9*d*., what would 56 lbs. cost ?.... *Ans*. 2*s*.

x. If 2 lbs. of tea cost 9*s*., what is the cost of 10 lbs.? ....*Ans*. 2*l*. 5*s*.

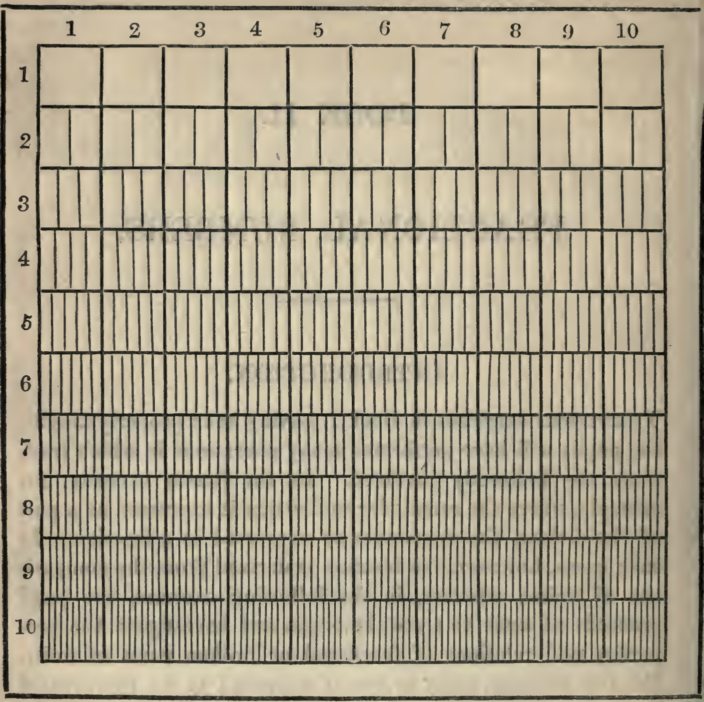
BOOK II.

FRACTIONAL NUMBERS.

INTRODUCTION.

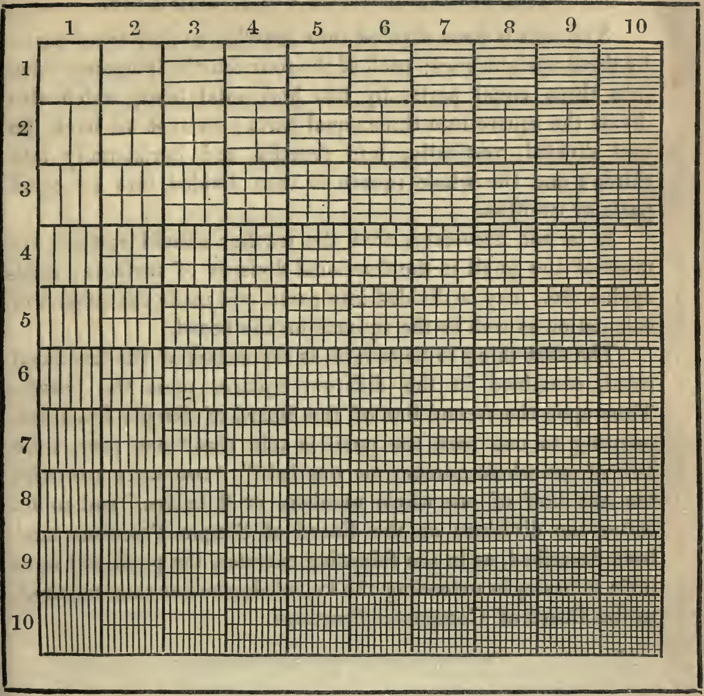
The various exercises in Book I. having been properly taught, the pupils will have performed many operations in which fractions are indirectly involved. In the fourth exercise, for example, where the remainder in division is expressed as a part of the divisor, the remainder so expressed is a fraction. In such cases, however, the fraction is derived from the comparison of units; whereas, in the following exercise, the pupil proceeds at once to trace the origin and investigate the properties and relations of fractional or broken parts of unity. For this purpose, unity or one is supposed to be represented by a square, and this square is divided into two, three, four, &c. equal parts, to form, respectively, the fractions one-half, one-third, one-fourth, &c. Thus, four-fifths of one or unity is represented by four of the five equal parts into which the square is divided; three-fourths by three of the four equal parts into which the square is divided; and so on, as the case may be.

We now replace the Board of Simple Units by two Boards, one of Simple and the other of Compound Fractions. The construction of these Boards will readily be understood by inspection, aided by the following explanation.



BOARD OF SIMPLE FRACTIONS.

The units or squares on the first line are undivided
The units or squares on the second line are divided into halves
The units or squares on the third line are divided into thirds
The units or squares on the fourth line are divided into fourths
The units or squares on the fifth line are divided into fifths
&c. &c. &c.



BOARD OF COMPOUND FRACTIONS.

The squares on this board are divided, by the upright lines, precisely in the same manner as on the Board of Simple Fractions. But in addition to this, each square in the second column is divided into two equal parts by a horizontal line; each square in the third column is divided into three equal parts by two horizontal lines; each square in the fourth column is divided into four equal parts by three horizontal lines; and so on.

To enable the teacher more fully to understand the manner in which these divisions are made, it will be sufficient to describe

The Third Square on the Fourth Line.

The unit is here divided into fourths, or four equal parts, by three upright lines; each of the four-fourths is again divided into three equal parts, by two horizontal lines, which also divide the square into three equal parts; so that we have the unit divided, vertically, into fourths, and horizontally into thirds; and the whole square is thus divided into 12 equal parts or twelfths.

It is not necessary, that the teacher should restrict the mind of the pupil to the fractional division of surfaces; rods, apples, &c., may be divided into parts, and made the subject of calculation as well as the squares on the board.

The first thing to be taught, is the names of the fractional parts, contained in the different squares upon the boards. Thus upon the second line of the Board of Simple Fractions, pointing to one part, the pupil is told that "one part is the half of one whole number, or one half," "two parts are twice the one half of one whole number, or 2 halves," and so on. After the divisions on the Board of Simple Fractions have been described, as in the following exercise, the pupil will have little difficulty in extending the knowledge he has thus acquired, to the Board of Compound Fractions.

FIRST EXERCISE.

Formation and Nature of Fractions.

This exercise is intended to exhibit the way in which a fraction is formed; to define the terms made use of, and to show upon what the magnitude of a fraction depends.

The Board of Simple Fractions will be required in this exercise.

Second Line, or Line of Halves.

Here unity is divided into two equal parts.

One part is the half of one, or one half.

Two parts are twice the half of one, or 2 halves.

Three parts are 3 times the half of one, or 3 halves.

&c. &c.

Third Line, or Line of Thirds.

Here unity is divided into three equal parts.

One part is the third of one whole number, or one third.

Two parts are twice the third of one whole number, or 2 thirds.

Three parts are 3 times the third of one whole number, or 3 thirds.

Four parts are 4 times the third of one whole number, or 4 thirds.

&c. &c.

Fourth Line, or Line of Fourths.

Here unity is divided into four equal parts.

One part is the fourth of one or one fourth.

Two parts are twice the fourth of one or 2 fourths.

Three parts are 3 times the fourth of one or 3 fourths.

Four parts are 4 times the fourth of one or 4 fourths.

Five parts are 5 times the fourth of one or 5 fourths.

&c. &c.

And so on to the fifth, sixth, seventh, &c. lines.

The pupils will now readily understand that every fraction is expressed by two numbers; the denominator, indicating the number of parts into which the unit is divided; and the numerator, showing how many of these parts are taken: for instance, in the fraction 3 fourths (which is also written $\frac{3}{4}$) 4 is the denominator, and 3 is the numerator.

Comparative Magnitude of Fractions having the same Denominator.

Second Line, or Line of Halves.

One half is less than 2 halves	2 halves are greater than one half
2 halves are less than 3 halves	3 halves are greater than 2 halves
3 halves are less than 4 halves	4 halves are greater than 3 halves
4 halves are less than 5 halves	5 halves are greater than 4 halves
&c.	&c.

Third Line, or Line of Thirds.

One third is less than 2 thirds	2 thirds are greater than 1 third
2 thirds are less than 3 thirds	3 thirds are greater than 2 thirds
&c.	&c.

Fourth Line, or Line of Fourths.

One fourth is less than 2 fourths	2 fourths are greater than 1 fourth
2 fourths are less than 3 fourths	3 fourths are greater than 2 fourths
&c.	&c.

Fifth Line, or Line of Fifths.

One fifth is less than 2 fifths	2 fifths are greater than 1 fifth
2 fifths are less than 3 fifths	3 fifths are greater than 2 fifths

And so on to the sixth, seventh, eighth, &c. Lines.

Comparative Magnitude of Fractions having different Denominators.

First Column.

One whole number is greater than one half	One half is less than one whole number
One half is greater than one third	One third is less than one half
One third is greater than one fourth	One fourth is less than one third
One fourth is greater than one fifth	One fifth is less than one fourth
One fifth is greater than one sixth	One sixth is less than one fifth
One sixth is greater than one seventh	One seventh is less than one sixth
One eighth is greater than one ninth	One ninth is less than one eighth
One ninth is greater than one tenth	One tenth is less than one ninth
&c. &c.	&c. &c.

The pupil will now readily understand: 1. That a fraction is increased, by increasing the numerator; for by doing so we increase the number of fractional parts taken; thus four-fifths are greater than three-fifths. 2. That as we increase the denominator, so we diminish the fraction; for by this means we divide the unit into more parts, and consequently each part will be rendered less, thus $\frac{3}{8}$ are less than $\frac{3}{4}$, (see the

eighth and fourth lines,) because the value of one-eighth is less than one-fourth. The converse of these propositions will also be easily understood by the pupil.

In the first part of this exercise the pupil has been taught another method of expressing a ratio, that is, by a fraction; for instance, instead of saying 3 times the fifth of a pound, he may now simply say 3-fifths ($\frac{3}{5}$) of a pound. The questions, therefore, belonging to the first case of ratios may be here proposed, only observing that such language as "3 times the fifth" is changed into "3-fifths," or $\frac{3}{5}$. In addition to these questions, the following may also be proposed.

Questions on the First Exercise.

i. Whether is $\frac{1}{2}$ or $\frac{1}{3}$ the greater?....*Ans.* $\frac{1}{2}$. Because $\frac{1}{2}$ is one of the two equal parts into which unity is divided; and $\frac{1}{3}$ is one of the three equal parts into which unity is divided.

ii. How many fifths are there in one?....*Ans.* 5-fifths.

iii. What do you mean by $\frac{3}{4}$?....*Ans.* That if unity be divided into four equal parts, three of these parts are $\frac{3}{4}$.

iv. Give another name for three fourths......*Ans.* Three times the fourth of one.

v. How many thirds are there in one?....*Ans.* Three thirds ($\frac{3}{3}$).

vi. How many thirds are there in 2?....*Ans.* $\frac{6}{3}$. Because 1 contains $\frac{3}{3}$, and therefore 2 will contain $\frac{3}{3}$ and $\frac{3}{3}$, which are $\frac{6}{3}$.

vii. How do you cut an apple so as to obtain $\frac{2}{3}$?....*Ans.* I cut the apple into 3 equal pieces; then 2 of these pieces will be $\frac{2}{3}$ of the whole apple.

viii. Why do the parts of this square represent fifths?.... *Ans.* Because the square, which we take as a unit, is divided into five equal parts; therefore one part will be the $\frac{1}{5}$ of the whole square.

ix. How many squares are there in the school-room window; and what part will 3 squares be of the whole window?.... *Ans.* There are 16 squares in the whole window: 3 squares will therefore be $\frac{3}{16}$ of the window.

Second Line, or Line of Halves.

i. 3 halves of a shilling contain how many pence?....*Ans.* 18*d.*; because the half of a shilling is 6*d.*, and therefore 3 halves will be 18*d.*

ii. What is the value of $\frac{5}{2}$ of a stone?....*Ans.* 35 lbs.; because the half of a stone is 7 lbs., and therefore $\frac{5}{2}$ will be 5 times 7 lbs., or 35 lbs.

Third Line, or Line of Thirds.

i. What is meant by $\frac{2}{3}$ of a shilling?....*Ans.* If 1*s.*, or 12*d.* be divided into three equal parts, then $\frac{2}{3}$ will be two of these parts.

ii. What is $\frac{5}{3}$ of 9*d.*?....*Ans.* 15*d.*; because the $\frac{1}{3}$ of 9*d.* is 3*d.*, and therefore $\frac{5}{3}$ will be 5 times 3*d.*, or 15*d.*

Fourth Line, or Line of Fourths.

i. $\frac{3}{4}$ of a pound are how many shillings?....*Ans.* 15s.; because the fourth of a pound is 5*s.*; and therefore $\frac{3}{4}$ will be 3 times 5*s.*, or 15*s.*

ii. Explain what is meant by $\frac{5}{4}$ of 16 feet.....*Ans.* That the fourth part of 16 feet, which is 4 feet, is to be repeated 5 times; which would give 20 feet for the value of $\frac{5}{4}$ of 16.

Fifth Line, or Line of Fifths.

i. How would you find $\frac{3}{5}$ of this rod?....*Ans.* By dividing it into 5 equal parts, (each of which would be a fifth,) and then taking 3 of these parts.

ii. Supposing the rod to be 10 feet long, what would be the length of $\frac{3}{5}$?....*Ans.* 6 feet; because $\frac{1}{5}$ would be 2 feet, and $\frac{3}{5}$ would be 3 times 2 feet, or 6 feet.

SECOND EXERCISE.

Reduction of Fractions.

The last exercise will have taught the pupil, that a fraction may be greater than unity; he will now be shown,—1st, how such fractions are converted into mixed numbers, that is, quantities which are made up of whole numbers and fractional parts; and 2nd, how these mixed numbers are converted into improper fractions, that is, fractions which are greater than unity.

In the present exercise, which requires the use of the Board of Simple Fractions only, the teacher proceeds thus. Suppose he has to show that five halves (an improper fraction) equal two units and a half (a mixed number). He places one of his pointers upon 5 halves, whilst with the other he refers to the 2 whole numbers, which enter into the composition of 5 halves: the pupil then says, "5 halves are 2 whole numbers and one half;" and conversely, "2 w. n. and one half are 5 halves." Pointing to 6 halves, in the same manner, the pupil says, "6 halves are 3 w. n.;" and then conversely, "3 w. n. are 6 halves." And so on to 7 halves, 8 halves, 9 halves, 10 halves, &c., &c. Such questions as the following may be put in the course of the exercise:

Teacher (pointing to the 5 halves upon the board). What sort of number is $\frac{5}{2}$?

Pupil. An improper fraction.

T. Why?

P. Because it is greater than unity.

T. How do you find the number of units contained in $\frac{5}{2}$?

P. As 1 whole number contains $\frac{2}{2}$, for every two halves I shall have one whole number; therefore $\frac{5}{2}$ are made up of $\frac{2}{2}$ and $\frac{2}{2}$ and $\frac{1}{2}$, that is 2 w. n. and $\frac{1}{2}$.

T. How many halves are there in $3\frac{1}{2}$?

P. 7 halves. Because 1 w. n. $= \frac{2}{2}$; and 3 w. n. $= \frac{2}{2} + \frac{2}{2} + \frac{2}{2}$, that is $\frac{6}{2}$ and therefore $3\frac{1}{2} = \frac{6}{2} + \frac{1}{2}$, or $\frac{7}{2}$.

Second Line, or Line of Halves.

2 halves are 1 whole number	1 w.n. contains 2 halves
3 halves are 1 w.n. and one half	1 w.n. and one half are 3 halves
4 halves are 2 w.n.	2 w.n. are 4 halves
5 halves are 2 w.n. and one half	2 w.n. and one half are 5 halves
6 halves are 3 w.n.	3 w.n. are 6 halves
7 halves are 3 w.n. and one half	3 w.n. and one half are 7 halves
8 halves are 4 w.n.	4 w.n. are 8 halves

And so on.

Third Line, or Line of Thirds.

3 thirds are 1 w.n.	1 w.n. contains 3 thirds
$\frac{4}{3}$* are 1 w.n. and $\frac{1}{3}$	1 w.n. and $\frac{1}{3}$ are $\frac{4}{3}$
$\frac{5}{3}$ are 1 w.n. and $\frac{2}{3}$	1 w.n. and $\frac{2}{3}$ are $\frac{5}{3}$
$\frac{6}{3}$ are 2 w.n.	2 w.n. are $\frac{6}{3}$
$\frac{7}{3}$ are 2 w.n. and $\frac{1}{3}$	2 w.n. and $\frac{1}{3}$ are $\frac{7}{3}$
$\frac{8}{3}$ are 2 w.n. and $\frac{2}{3}$	2 w.n. and $\frac{2}{3}$ are $\frac{8}{3}$
$\frac{9}{3}$ are 3 w.n.	3 w.n. are $\frac{9}{3}$

And so on.

Fourth Line, or Line of Fourths.

4 fourths are 1 w.n.	1 w.n. contains 4 fourths
$\frac{5}{4}$ are 1 w.n. and $\frac{1}{4}$	1 w.n. and $\frac{1}{4}$ are $\frac{5}{4}$
$\frac{6}{4}$ are 1 w.n. and $\frac{2}{4}$	1 w.n. and $\frac{2}{4}$ are $\frac{6}{4}$
$\frac{7}{4}$ are 1 w.n. and $\frac{3}{4}$	1 w.n. and $\frac{3}{4}$ are $\frac{7}{4}$
$\frac{8}{4}$ are 2 w.n.	2 w.n. are $\frac{8}{4}$
$\frac{9}{4}$ are 2 w.n. and $\frac{1}{4}$	2 w.n. and $\frac{1}{4}$ are $\frac{9}{4}$

And so on.

Fifth Line, or Line of Fifths.

5 fifths are 1 w.n.	1 w.n. contains 5 fifths
$\frac{6}{5}$ are 1 w.n. and $\frac{1}{5}$	1 w.n. and $\frac{1}{5}$ are $\frac{6}{5}$
$\frac{7}{5}$ are 1 w.n. and $\frac{2}{5}$	1 w.n. and $\frac{2}{5}$ are $\frac{7}{5}$
$\frac{8}{5}$ are 1 w.n. and $\frac{3}{5}$	1 w.n. and $\frac{3}{5}$ are $\frac{8}{5}$
$\frac{9}{5}$ are 1 w.n. and $\frac{4}{5}$	1 w.n. and $\frac{4}{5}$ are $\frac{9}{5}$
$\frac{10}{5}$ are 2 w.n.	2 w.n. are $\frac{10}{5}$
$\frac{11}{5}$ are 2 w.n. and $\frac{1}{5}$	2 w.n. and $\frac{1}{5}$ are $\frac{11}{5}$

And so on.

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* Such lines as this are to be read thus: "Four-thirds are one whole number and one-third."

Sixth Line, or Line of Sixths.

$\frac{6}{6}$ are 1 w.n.	1 w.n. contains $\frac{6}{6}$
$\frac{7}{6}$ are 1 w.n. and $\frac{1}{6}$	1 w.n. and $\frac{1}{6}$ are $\frac{7}{6}$
$\frac{8}{6}$ are 1 w.n. and $\frac{2}{6}$	1 w.n. and $\frac{2}{6}$ are $\frac{8}{6}$
$\frac{9}{6}$ are 1 w.n. and $\frac{3}{6}$	1 w.n. and $\frac{3}{6}$ are $\frac{9}{6}$
$\frac{10}{6}$ are 1 w.n. and $\frac{4}{6}$	1 w.n. and $\frac{4}{6}$ are $\frac{10}{6}$

And so on.

The teacher is to proceed in the same way with the seventh, eighth, ninth, and tenth lines.

This exercise, in general, teaches, that when the numerator of a fraction is greater than its denominator, then such fraction is greater than unity, and that the whole numbers are extracted, by dividing the numerator by the denominator for the units, and then annexing the remainder in the form of a fraction for the complete value, thus, $\frac{17}{5}=3\frac{2}{5}$; and that conversely the mixed number $3\frac{2}{5}$ is reduced to a fraction, by converting the 3 whole numbers into fifths, by multiplying by 5, and then adding this product to the numerator of the fractional part for the number of fifths in the proposed number.

Questions on the Second Exercise.

Second Line, or Line of Halves.

i. In $2\frac{1}{2}$, how many halves?....*Ans.* $\frac{5}{2}$, because 2 w. n. contain 4 halves, and therefore 2 w. n. and $\frac{1}{2}$ will contain $\frac{4}{2}$ and $\frac{1}{2}$, which equal $\frac{5}{2}$.

ii. How many whole numbers are there in 17 halves?.... *Ans.* $8\frac{1}{2}$.

Proof. 16 halves are 8 w. n., and 17 halves are $8\frac{1}{2}$.

iii. Reduce $3\frac{1}{2}$ to halves....*Ans.* $\frac{7}{2}$, because 1 contains $\frac{2}{2}$, therefore 3 w. n. will contain $\frac{2}{2}$, and $\frac{2}{2}$, and $\frac{2}{2}$, which are $\frac{6}{2}$, therefore $3\frac{1}{2}=\frac{6}{2}+\frac{1}{2}$, that is, $\frac{7}{2}$.

Third Line, or Line of Thirds.

i. Reduce $3\frac{1}{3}$ to thirds.....*Ans.* $\frac{10}{3}$, because $1=\frac{3}{3}$; and $3=\frac{3}{3}$ repeated 3 times, which are $\frac{9}{3}$, therefore $3\frac{1}{3}=\frac{9}{3}+\frac{1}{3}$, or $\frac{10}{3}$.

ii. Reduce 13 thirds to whole numbers.....*Ans.* $4\frac{1}{3}$

iii. Reduce 23 thirds to whole numbers.....*Ans.* $7\frac{2}{3}$.

Fourth Line, or Line of Fourths.

i. In $4\frac{1}{4}$, how many fourths?....*Ans.* $\frac{17}{4}$.

ii. In $5\frac{3}{4}$, how many fourths?....*Ans.* $\frac{23}{4}$.

iii. Reduce $\frac{29}{4}$ to whole numbers.....*Ans.* $7\frac{1}{4}$.

Tenth Line, or Line of Tenths.

i. Reduce $8\frac{3}{10}$ to tenths.....*Ans.* $\frac{83}{10}$.

ii. Reduce $5\frac{4}{10}$ to tenths.....*Ans.* $\frac{54}{10}$.

iii. Reduce $\frac{73}{10}$ to whole numbers.....*Ans.* $7\frac{3}{10}$.

iv. Reduce $\frac{37}{10}$ to whole numbers.....*Ans.* $3\frac{7}{10}$.

Miscellaneous.

i. Reduce $4\frac{2}{4}$ to fourths.....*Ans.* $\frac{18}{4}$.

ii. Reduce $7\frac{4}{6}$ to sixths.....*Ans.* $\frac{46}{6}$.

iii. Reduce $2\frac{6}{8}$ to eighths.....*Ans.* $\frac{22}{8}$.

THIRD EXERCISE.

Addition and Subtraction of Fractions having a Common Denominator.

In this exercise the pupil is led to perform the same operations upon fractional units, or spaces representing certain parts of unity, that he has already performed upon simple units in the First Exercise of Book I. For instance, he is shown that 3 fourths added to 5 fourths produce 8 fourths, in the same way that 3 units added to 5 units produce 8 units.

The Board of Simple Fractions is the only one required for this exercise, and in teaching it the pointers are to be placed so as to direct the attention of the pupils to the fractions to be added or subtracted. Such questions as the following may be put in the course of the exercise.

Teacher. What *kind* of fractions have you been adding?

Pupil. Those that are composed of the same parts of unity.

T. How do you obtain the addition of $\frac{5}{2}$ and $\frac{4}{2}$?

P. By adding 5 and 4, which gives $\frac{9}{2}$. Because 5 parts and 4 parts produce 9 parts.

T. Why can you not add $\frac{1}{5}$ and $\frac{1}{6}$ in the same manner?

P. Because $\frac{1}{5}$ and $\frac{1}{6}$ are different parts of unity.

T. What must be done in order to add these fractions?

P. They must be brought to the same part of unity.

Third Line, or Line of Thirds.

$\frac{1}{3}$ and $\frac{1}{3}$ make $\frac{2}{3}$	$\frac{1}{3}$ from $\frac{2}{3}$ and $\frac{1}{3}$ remains
$\frac{2}{3}$ and $\frac{1}{3}$ make $\frac{3}{3}$ or 1	$\frac{1}{3}$ from $\frac{3}{3}$ and $\frac{2}{3}$ remain
*$\frac{3}{3}$ and $\frac{1}{3}$ make $\frac{4}{3}$ or $1\frac{1}{3}$	$\frac{1}{3}$ from $\frac{4}{3}$ and $\frac{3}{3}$ or 1 remain
$\frac{4}{3}$ and $\frac{1}{3}$ make $\frac{5}{3}$ or $1\frac{2}{3}$	$\frac{1}{3}$ from $\frac{5}{3}$ and $\frac{4}{3}$ or $1\frac{1}{3}$ remain
And so on.	And so on.

* Read "Three-thirds and one-third make four-thirds or one and one-third," &c.

$\frac{1}{3}$ and $\frac{2}{3}$ make $\frac{3}{3}$ or 1	$\frac{2}{3}$ from $\frac{3}{3}$ and $\frac{1}{3}$ remains
$\frac{2}{3}$ and $\frac{2}{3}$ make $\frac{4}{3}$ or $1\frac{1}{3}$	$\frac{2}{3}$ from $\frac{4}{3}$ and $\frac{2}{3}$ remain
$\frac{3}{3}$ and $\frac{2}{3}$ make $\frac{5}{3}$ or $1\frac{2}{3}$	$\frac{2}{3}$ from $\frac{5}{3}$ and $\frac{3}{3}$ or 1 remains
$\frac{4}{3}$ and $\frac{2}{3}$ make $\frac{6}{3}$ or 2	$\frac{2}{3}$ from $\frac{6}{3}$ and $\frac{4}{3}$ or $1\frac{1}{3}$ remain
And so on.	And so on.

$\frac{1}{3}$ and $\frac{3}{3}$ make $\frac{4}{3}$ or $1\frac{1}{3}$	$\frac{3}{3}$ from $\frac{4}{3}$ and $\frac{1}{3}$ remains
$\frac{2}{3}$ and $\frac{3}{3}$ make $\frac{5}{3}$ or $1\frac{2}{3}$	$\frac{3}{3}$ from $\frac{5}{3}$ and $\frac{2}{3}$ remain
$\frac{3}{3}$ and $\frac{3}{3}$ make $\frac{6}{3}$ or 2	$\frac{3}{3}$ from $\frac{6}{3}$ and $\frac{3}{3}$ or 1 remains
$\frac{4}{3}$ and $\frac{3}{3}$ make $\frac{7}{3}$ or $2\frac{1}{3}$	$\frac{3}{3}$ from $\frac{7}{3}$ and $\frac{4}{3}$ or $1\frac{1}{3}$ remain
And so on.	And so on.

Then follows the addition by $\frac{4}{3}$, $\frac{5}{3}$, $\frac{6}{3}$, $\frac{7}{3}$, &c.

The table here exhibited of the line of thirds, is to be extended to the lines of halves, fourths, fifths, &c.

This exercise having been sufficiently repeated, the pupils will be prepared to understand, that *we add or subtract fractions of the same denomination, by taking the sum or difference of their numerators, and then placing the common denominator beneath this sum or difference.* In fact $\frac{2}{5}$ and $\frac{4}{5}$ make $\frac{6}{5}$, in the same manner, that two quantities and four quantities of the *same kind,* make six quantities of this kind, whatever these quantities may be. In like manner, two quantities, of any kind, taken from four quantities, of the same kind, leave two quantities of the same kind.

When the fractions have different denominators, we cannot add them in that state, because the parts are of different magnitudes.

Questions on the Third Exercise.

Second Line, or Line of Halves.

i. What is the sum of $\frac{1}{2}$ and $\frac{3}{2}$?....*Ans.* $\frac{4}{2} = 2$.

ii. What is the sum of $\frac{5}{2}$ and $\frac{2}{2}$?....*Ans.* $\frac{7}{2} = 3\frac{1}{2}$.

iii. What is the sum of $\frac{5}{2}$ and $\frac{3}{2}$?....*Ans.* $\frac{8}{2} = 4$.

iv. What is the difference between $\frac{5}{2}$ and $\frac{3}{2}$?....*Ans.* $\frac{2}{2} = 1$.

v. What is the difference between $\frac{11}{2}$ and $\frac{5}{2}$?....*Ans.* $\frac{6}{2} = 3$.

vi. What is the difference between $\frac{13}{2}$ and $\frac{3}{2}$?....*Ans.* $\frac{10}{2} = 5$.

vii. How many times is $\frac{1}{2}$ repeated to make $\frac{5}{2}$?....*Ans.* 5 times.

Third Line, or Line of Thirds.

i. What is the sum of $\frac{1}{3}$ and $\frac{5}{3}$?....*Ans.* $\frac{6}{3}$ or 2.

ii. What is the sum of $\frac{5}{3}$ and $\frac{6}{3}$?....*Ans.* $\frac{11}{3} = 3\frac{2}{3}$.

iii. What is the sum of $\frac{7}{3}$ and $\frac{1}{3}$?....*Ans.* $\frac{8}{3} = 2\frac{2}{3}$.

iv. What is the sum of $2\frac{2}{3}$ and $3\frac{1}{3}$?....*Ans.* $\frac{18}{3} = 6$.

v. What is the difference between $\frac{4}{3}$ and $\frac{2}{3}$?....*Ans.* $\frac{2}{3}$.

vi. What is the difference between $\frac{8}{3}$ and $\frac{5}{3}$?....*Ans.* $\frac{3}{3} = 1$.

vii. What is the difference between $2\frac{1}{3}$ and $\frac{4}{3}$?....*Ans.* $\frac{3}{3} = 1$.

Because $2\frac{1}{3} = \frac{7}{3}$ and $\frac{7}{3} - \frac{4}{3} = \frac{3}{3}$ or 1.

viii. How many times must $\frac{1}{3}$ be repeated to make $\frac{4}{3} + \frac{2}{3}$? ....*Ans.* 6 times: because $\frac{4}{3} + \frac{2}{3} = \frac{6}{3}$.

ix. If I divide an apple into 3 equal parts, and give one of them away, how much of the apple have I left?....*Ans.* $\frac{2}{3}$.

Fourth Line, or Line of Fourths.

i. What is the sum of $\frac{3}{4}$ and $\frac{2}{4}$?....*Ans.* $\frac{5}{4}$ or $1\frac{1}{4}$.

ii. What is the sum of $\frac{1}{4}$ and $\frac{5}{4}$?....*Ans.* $\frac{6}{4}$ or $1\frac{2}{4}$.

iii. What is the sum of $2\frac{1}{4}$ and $\frac{3}{4}$?....*Ans.* $\frac{12}{4}$ or 3.

Because $2\frac{1}{4} = \frac{9}{4}$, then $\frac{9}{4} + \frac{3}{4} = \frac{12}{4}$ or 3.

iv. How many times must $\frac{1}{4}$ be repeated to make $\frac{5}{4}$?.... *Ans.* 5 times.

v. What is the difference between $\frac{6}{4}$ and $\frac{3}{4}$?....*Ans.* $\frac{3}{4}$.

vi. What is the difference between $\frac{5}{4}$ and $\frac{2}{4}$?....*Ans.* $\frac{3}{4}$.

vii. What is the difference between $2\frac{3}{4}$ and $\frac{2}{4}$?....*Ans.* $\frac{9}{4}$ or $2\frac{1}{4}$.

Because $2\frac{3}{4} = \frac{11}{4}$ and $\frac{11}{4} - \frac{2}{4} = \frac{9}{4}$ or $2\frac{1}{4}$.

Fifth Line, or Line of Fifths.

i. A grocer divided a cheese into five equal parts, and sold three of them; how much of the cheese had he left? ....*Ans.* $\frac{2}{5}$.

ii. A post is two-fifths of its length in the ground, how much of its length stands above ground?....*Ans.* $\frac{3}{5}$.

iii. What is the sum of $\frac{2}{5}$ and $\frac{1}{5}$?....*Ans.* $\frac{3}{5}$.

iv. What is the sum of $\frac{4}{5}$ and $\frac{8}{5}$?....*Ans.* $\frac{12}{5}$ or $2\frac{2}{5}$.

v. What is the sum of $2\frac{2}{5}$ and $\frac{6}{5}$?....*Ans.* $\frac{18}{5}$ or $3\frac{3}{5}$.

Because $2\frac{2}{5} = \frac{12}{5}$ and $\frac{12}{5} + \frac{6}{5} = \frac{18}{5}$ or $3\frac{3}{5}$.

vi. How many times must $\frac{1}{5}$ be repeated to make $\frac{8}{5}$?.... *Ans.* 8 times.

vii. What is the difference between $\frac{3}{5}$ and $\frac{1}{5}$?....*Ans.* $\frac{2}{5}$.

viii. What is the difference between $2\frac{4}{5}$ and $\frac{7}{5}$?....*Ans.* $\frac{7}{5}$.

Because $2\frac{4}{5} = \frac{14}{5}$ and $\frac{14}{5} - \frac{7}{5} = \frac{7}{5}$.

ix. How many times must $\frac{4}{5}$ be repeated to make $\frac{20}{5}$?.... *Ans.* 5 times.

Sixth Line, or Line of Sixths.

i. What is the sum of $\frac{2}{6}$ and $\frac{5}{6}$?....*Ans.* $\frac{7}{6}$ or $1\frac{1}{6}$.

ii. What is the sum of $\frac{5}{6}$ and $\frac{10}{6}$?....*Ans.* $\frac{15}{6}$ or $2\frac{3}{6}$.

iii. What is the sum of $5\frac{1}{6}$ and $\frac{1}{6}$?....*Ans.* $\frac{32}{6}$ or $5\frac{2}{6}$.

Because by adding the fractions we have $\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$, which added to 5 gives $5\frac{2}{6}$.

iv. What number must be added to $1\frac{5}{6}$, so that the sum may be $4\frac{1}{6}$?....*Ans.* $2\frac{2}{6}$.

Because as the number required added to $1\frac{5}{6} = 4\frac{1}{6}$, the difference between $4\frac{1}{6}$ and $1\frac{5}{6}$ will give the number required: hence $4\frac{1}{6} = \frac{25}{6}$, $1\frac{5}{6} = \frac{11}{6}$, $\frac{25}{6} - \frac{11}{6} = \frac{14}{6}$ or $2\frac{2}{6}$.

v. A person sold $\frac{5}{6}$ of his property, what part had he left?....*Ans.* $\frac{1}{6}$.

FOURTH EXERCISE.

Multiplication and Division of Fractions by a whole number.

In this exercise the pupil is led to perform the same operations upon fractional units that he has already performed upon simple units in the Second Exercise of Book I. In all the examples of this exercise the multiplier of the fraction is a whole number; and the fractional divisor is contained a certain number of times, without a remainder, in the dividend. Other varieties of multiplication and division will be given in the Sixth and Seventh Exercises.

The teacher may illustrate this exercise on any one of the lines of the Board of Simple Fractions. Suppose he select the fifth line. When he points off $\frac{4}{5}$, or four fifths, with one rod, and $\frac{2}{5}$, or two fifths, with the other, the pupil says "twice $\frac{2}{5}$ are $\frac{4}{5}$;" and conversely, "$\frac{2}{5}$ are contained in $\frac{4}{5}$ twice;"—"the half of $\frac{4}{5}$ is $\frac{2}{5}$," and so on. In such an instance as this the right hand pointer is removed the space of $\frac{2}{5}$, at each step of the table, whilst the other rod is used to show the number of times that $\frac{2}{5}$ are repeated.

Such questions as the following may be put in the course of the exercise.

Teacher. What do you understand by 3 times $\frac{2}{5}$?

Pupil. That $\frac{2}{5}$ are repeated 3 times, that is, $\frac{2}{5}$ and $\frac{2}{5}$ and $\frac{2}{5} = \frac{6}{5}$.

Teacher. What is meant by $\frac{2}{5}$ being contained in $\frac{6}{5}$ three times?

Pupil. That two-fifths can be taken out of $\frac{6}{5}$ three times.

Teacher. How do you find the third of $\frac{6}{5}$?

Pupil. If $\frac{6}{5}$ be divided into three equal parts, one of these parts is the third of $\frac{6}{5}$; thus $\frac{2}{5}$ and $\frac{2}{5}$ and $\frac{2}{5}$ or 3 times $\frac{2}{5} = \frac{6}{5}$; therefore $\frac{2}{5}$ are the third of $\frac{6}{5}$.

Teacher. Comparing this exercise with the second exercise on unity, do you find any difference between the multiplication of units, and the multiplication of fractions by whole numbers?

Pupil. The only difference is, that in the one case the results are units, and in the other fractional parts.

Teacher. 3 times $\frac{2}{3}$ being $\frac{6}{3}$, how do you obtain the product of 4 times $\frac{2}{3}$?

Pupil. By increasing $\frac{6}{3}$ by $\frac{2}{3}$ which make $\frac{8}{3}$.

In this exercise the first column is to be recited by itself; then the first and second columns; and lastly the first and third columns.

Second Line, or Line of Halves.

First Column.	*Second Column.*	*Third Column.*
Once one-half is $\frac{1}{2}$	$\frac{1}{2}$ is contained in $\frac{1}{2}$ once	
Twice $\frac{1}{2}$ are 2 halves or 1	$\frac{1}{2}$ is contained in $\frac{2}{2}$ twice	the half of $\frac{2}{2}$ is $\frac{1}{2}$
3 times $\frac{1}{2}$ are 3 halves or $1\frac{1}{2}$	$\frac{1}{2}$ is contained in $\frac{3}{2}$ 3 times	the third of $\frac{3}{2}$ is $\frac{1}{2}$
4 × $\frac{1}{2}$ are 4 halves or 2	$\frac{1}{2}$ is contained in $\frac{4}{2}$ 4 times	the fourth of $\frac{4}{2}$ is $\frac{1}{2}$
5 × $\frac{1}{2}$ are 5 halves r $2\frac{1}{2}$	$\frac{1}{2}$ is contained in $\frac{5}{2}$ 5 times	the fifth of $\frac{5}{2}$ is $\frac{1}{2}$
And so on.	And so on.	And so on.

Once $\frac{3}{2}$ is 3 halves	$\frac{3}{2}$ are contained in $\frac{3}{2}$ once	
Twice $\frac{3}{2}$ are 6 halves or 3	$\frac{3}{2}$ are contained in $\frac{6}{2}$ twice	the half of $\frac{6}{2}$ is $\frac{3}{2}$ or $1\frac{1}{2}$
3 times $\frac{3}{2}$ are $\frac{9}{2}$ or $4\frac{1}{2}$	$\frac{3}{2}$ are contained in $\frac{9}{2}$ 3 times	the third of $\frac{9}{2}$ is $\frac{3}{2}$ or $1\frac{1}{2}$
And so on.	And so on.	And so on.

And so on to the Multiplication of any number of halves.

In like manner we proceed with any other line, such for example as the

Fifth Line, or Line of Fifths.

First Column.	*Second Column.*	*Third Column.*
Once $\frac{1}{5}$ is $\frac{1}{5}$	$\frac{1}{5}$ is contained in $\frac{1}{5}$ once	
Twice $\frac{1}{5}$ are $\frac{2}{5}$	$\frac{1}{5}$ is contained in $\frac{2}{5}$ twice	the half of $\frac{2}{5}$ is $\frac{1}{5}$
3 times $\frac{1}{5}$ are $\frac{3}{5}$	$\frac{1}{5}$ is contained in $\frac{3}{5}$ 3 times	the third of $\frac{3}{5}$ is $\frac{1}{5}$
4 times $\frac{1}{5}$ are $\frac{4}{5}$	$\frac{1}{5}$ is contained in $\frac{4}{5}$ 4 times	the fourth of $\frac{4}{5}$ is $\frac{1}{5}$
And so on.	And so on.	And so on.
Once $\frac{2}{5}$ is $\frac{2}{5}$	$\frac{2}{5}$ are contained in $\frac{2}{5}$ once	
Twice $\frac{2}{5}$ are $\frac{4}{5}$	$\frac{2}{5}$ are contained in $\frac{4}{5}$ twice	the half of $\frac{4}{5}$ is $\frac{2}{5}$
$3 \times \frac{2}{5}$ are $\frac{6}{5}$ or $1\frac{1}{5}$	$\frac{2}{5}$ are contained in $\frac{6}{5}$ 3 times	the third of $\frac{6}{5}$ is $\frac{2}{5}$
$4 \times \frac{2}{5}$ are $\frac{8}{5}$ or $1\frac{3}{5}$	$\frac{2}{5}$ are contained in $\frac{8}{5}$ 4 times	the fourth of $\frac{8}{5}$ is $\frac{2}{5}$
And so on.	And so on.	And so on.
Once $\frac{3}{5}$ is $\frac{3}{5}$	$\frac{3}{5}$ are contained in $\frac{3}{5}$ once	
Twice $\frac{3}{5}$ are $\frac{6}{5}$ or $1\frac{1}{5}$	$\frac{3}{5}$ are contained in $\frac{6}{5}$ twice	the half of $\frac{6}{5}$ is $\frac{3}{5}$
$3 \times \frac{3}{5}$ are $\frac{9}{5}$ or $1\frac{4}{5}$	$\frac{3}{5}$ are contained in $\frac{9}{5}$ 3 times	the third of $\frac{9}{5}$ is $\frac{3}{5}$
$4 \times \frac{3}{5}$ are $\frac{12}{5}$ or $2\frac{2}{5}$	$\frac{3}{5}$ are contained in $\frac{12}{5}$ 4 times	the fourth of $\frac{12}{5}$ is $\frac{3}{5}$
And so on.	And so on.	And so on.
Once $\frac{4}{5}$ is $\frac{4}{5}$	$\frac{4}{5}$ are contained in $\frac{4}{5}$ once	
Twice $\frac{4}{5}$ are $\frac{8}{5}$ or $1\frac{3}{5}$	$\frac{4}{5}$ are contained in $\frac{8}{5}$ twice	the half of $\frac{8}{5}$ is $\frac{4}{5}$
$3 \times \frac{4}{5}$ are $\frac{12}{5}$ or $2\frac{2}{5}$	$\frac{4}{5}$ are contained in $\frac{12}{5}$ 3 times	the third of $\frac{12}{5}$ is $\frac{4}{5}$
And so on.	And so on.	And so on.

And so on to the product of any number of fifths.

This exercise shows that *we multiply a fraction by a whole number when we multiply the numerator by that whole number:* thus to multiply $\frac{3}{4}$ by 5, the multiplicand $\frac{3}{4}$ must be added 5 times, or $\frac{3}{4}$ taken 5 times $= \frac{15}{4}$. The inverse part of this exercise also shows, that *we divide a fraction by a whole number, when we divide the numerator by that whole number:* thus $\frac{3}{4}$ is the quotient of $\frac{15}{4}$ divided by 5; or the fifth of $\frac{15}{4}$ is $\frac{3}{4}$.

It is important also to observe, that any fraction multiplied by its denominator gives the numerator, that is, $\frac{5}{7} \times 7 = 5$.

Questions on the Fourth Exercise.

Second Line, or Line of Halves.

i. How much is 6 times $\frac{3}{2}$?....*Ans.* $\frac{18}{2}$ or 9 whole numbers.

ii. What is 4 times $\frac{5}{2}$?....*Ans.* $\frac{20}{2}$ or 10 whole numbers.

iii. How often are $\frac{3}{2}$ contained in $\frac{18}{2}$?....*Ans.* 6 times.

iv. What is the fifth part of $\frac{15}{2}$?....*Ans.* $\frac{3}{2}$.

Because $\frac{15}{2}$ divided into 5 equal parts, gives us $\frac{3}{2}$ for one of those parts.

v. What is meant by $\frac{3}{2}$ multiplied by 4?....*Ans.* $\frac{3}{2}$ repeated 4 times; that is $\frac{3}{2}+\frac{3}{2}+\frac{3}{2}+\frac{3}{2}=\frac{12}{2}$ or 6.

vi. If $\frac{7}{2}$ be multiplied by 3, what is the product?....*Ans.* $\frac{21}{2}$ or $10\frac{1}{2}$.

Because $\frac{7}{2}$ repeated 3 times give $\frac{21}{2}$ or $10\frac{1}{2}$.

Third Line, or Line of Thirds.

i. Multiply $\frac{4}{3}$ by 6.....*Ans.* The product is $\frac{24}{3}$ or 8 whole numbers.

ii. When $\frac{5}{3}$ are repeated 4 times, what is the result?.... *Ans.* 4 times $\frac{5}{3}$ are $\frac{20}{3}$ or $6\frac{2}{3}$.

iii. What is the third part of $\frac{18}{3}$?....*Ans.* $\frac{6}{3}$ or 2.

Because when $\frac{18}{3}$ are divided into 3 equal parts, each of these parts will be $\frac{6}{3}$ or 2 whole numbers.

iv. What is 7 times $\frac{2}{3}$?....*Ans.* $\frac{14}{3}$ or $4\frac{2}{3}$.

v. How many oranges should I require so as to be able to give two-thirds of an orange to each of 4 boys?....*Ans.* $2\frac{2}{3}$.

Fourth Line, or Line of Fourths.

i. How much is 8 times $\frac{3}{4}$?....*Ans.* $\frac{24}{4}$ or 6 whole numbers.

ii. Divide $\frac{36}{4}$ into 6 equal parts, what will each of those parts be?....*Ans.* $\frac{6}{4}$ or $1\frac{2}{4}$.

iii. What is 7 times $\frac{2}{4}$?....*Ans.* $\frac{14}{4}$ or $3\frac{2}{4}$.

Because $\frac{2}{4}$ repeated 7 times are $\frac{14}{4}$ or $3\frac{2}{4}$

iv. How often must $\frac{3}{4}$ be repeated to make $\frac{15}{4}$?....*Ans.* 5 times.

Fifth Line, or Line of Fifths.

i. Multiply $\frac{3}{5}$ by 6.....*Ans.* The product is $\frac{18}{5}$ or $3\frac{3}{5}$.

ii. Find the product when $\frac{4}{5}$ are multiplied by 5. ....*Ans.* $\frac{20}{5}$ or 4 whole numbers.

iii. How often must $\frac{2}{5}$ be repeated to make up $\frac{12}{5}$?....*Ans.* 6 times.

iv. Divide $\frac{18}{5}$ into 3 equal parts.....*Ans.* $\frac{6}{5}$ or $1\frac{1}{5}$.
Because $\frac{6}{5} + \frac{6}{5} + \frac{6}{5}$, or 3 times $\frac{6}{5} = \frac{18}{5}$.

v. How many $\frac{3}{5}$ can be taken out of $\frac{9}{5}$?....*Ans.* 3.
Because $\frac{3}{5} + \frac{3}{5} + \frac{3}{5} = \frac{9}{5}$, that is, $\frac{9}{5}$ is made up of three $\frac{3}{5}$.

vi. Divide 2 apples and $\frac{2}{5}$ of an apple equally amongst 3 boys.....*Ans.* Each boy will receive $\frac{4}{5}$ of an apple.

Because each apple contains 5 fifths, therefore 2 apples and $\frac{2}{5}$ will contain $\frac{12}{5}$, and then if these $\frac{12}{5}$ be divided amongst 3 boys, each boy will receive the third of $\frac{12}{5}$ or $\frac{4}{5}$.

vii. Eight loaves of bread are divided each into 5 equal parts, how can I distribute them equally amongst 10 poor persons?....*Ans.* By giving to each person $\frac{4}{5}$ of a loaf.

FIFTH EXERCISE.

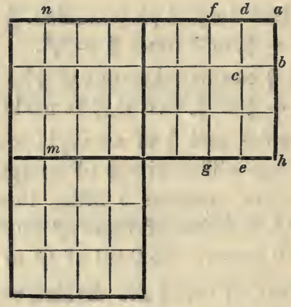
Reduction of Fractions to a Common Denominator.

In the first exercise of this Book it was shown that a fraction becomes greater when the numerator is increased, and less when the denominator is increased. The present exercise shows how the denominator of a fraction may be changed without altering the magnitude of the fraction. This operation is important, because it enables the pupil so to adjust two or more fractions that they may have the same denominator, in which state they may be more conveniently added or compared.

This exercise requires the use of the Compound Board.

When the teacher is about to introduce any square for the first time he is recommended to construct the figure on a large scale upon the black board in the following manner:

The Third Square of the Fourth Line on the Board of Compound Fractions.



The vertical lines in the figure are first drawn, and the pupils are shown that each square is thus divided into fourths. Each fourth is then divided into three equal parts by the horizontal lines; so that the pupils cannot fail to observe that each compartment, *a, b, c, d,* thus formed, is the third part of one-fourth, and at the same time one-twelfth of the whole square or unit. By means of this figure *fourths* are reduced to twelfths. When *thirds* are required to be reduced to twelfths, the squares must be extended in the vertical direction; and each square being first divided by the horizontal lines into 3 equal parts, each third, thus formed, is divided by the vertical lines into 4 equal parts.

The exercise on this square commences with the reduction of fourths to twelfths, and conversely; and concludes with the reduction of thirds to twelfths. When the construction of the figure has been described by the teacher, the pupil says, "1 is $\frac{12}{12}$;" and conversely, "$\frac{12}{12}$ are 1." The teacher now places his rod along the upright line *d e*, and then the pupil says "$\frac{1}{4}$ is $\frac{3}{12}$;" and conversely, "$\frac{3}{12}$ are $\frac{1}{4}$." The pointer is now removed to the line *f g;* and the pupil says, "$\frac{2}{4}$ are twice

$\frac{3}{12}$ or $\frac{6}{12}$;" and conversely, "$\frac{6}{12}$ are twice $\frac{3}{12}$ or $\frac{2}{4}$:" when the pointer is laid upon the line *d e*, the pupil says, "$\frac{7}{4}$ are 7 times $\frac{3}{12}$ or $\frac{21}{12}$;" and conversely, "$\frac{21}{12}$ are 7 times $\frac{3}{12}$ or $\frac{7}{4}$:" and so on. For the reduction of thirds to twelfths, the teacher places his rod on the first horizontal line dividing the square, the pupil then says, "$\frac{1}{3}$ is $\frac{4}{12}$;" and conversely, "$\frac{4}{12}$ are $\frac{1}{3}$:" the pointer being removed to the next horizontal line, the pupil says, "$\frac{2}{3}$ are twice $\frac{4}{12}$ or $\frac{8}{12}$;" and so on.

Second Line of Squares.

Second Square,

in which halves are reduced to fourths; and conversely.

1 is $\frac{4}{4}$	$\frac{4}{4}$ are 1
$\frac{1}{2}$ is $\frac{2}{4}$	$\frac{2}{4}$ are $\frac{1}{2}$
$\frac{2}{2}$ are twice $\frac{2}{4}$ or $\frac{4}{4}$	$\frac{4}{4}$ are twice $\frac{2}{4}$ or $\frac{2}{2}$
$\frac{3}{2}$ are 3 times $\frac{2}{4}$ or $\frac{6}{4}$	$\frac{6}{4}$ are 3 times $\frac{2}{4}$ or $\frac{3}{2}$
$\frac{4}{2}$ are $4 \times \frac{2}{4}$ or $\frac{8}{4}$	$\frac{8}{4}$ are $4 \times \frac{2}{4}$ or $\frac{4}{2}$
&c.	&c.

Third Square,

in which halves and thirds are reduced to the same denominator, viz. to sixths; and conversely.

1 is $\frac{6}{6}$	$\frac{6}{6}$ are 1
$\frac{1}{2}$ is $\frac{3}{6}$	$\frac{3}{6}$ are $\frac{1}{2}$
$\frac{2}{2}$ are twice $\frac{3}{6}$ or $\frac{6}{6}$	$\frac{6}{6}$ are twice $\frac{3}{6}$ or $\frac{2}{2}$
$\frac{3}{2}$ are 3 times $\frac{3}{6}$ or $\frac{9}{6}$	$\frac{9}{6}$ are 3 times $\frac{3}{6}$ or $\frac{3}{2}$
$\frac{4}{2}$ are $4 \times \frac{3}{6}$ or $\frac{12}{6}$	$\frac{12}{6}$ are $4 \times \frac{3}{6}$ or $\frac{4}{2}$
&c.	&c.

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1 is $\frac{6}{6}$	$\frac{6}{6}$ are 1
$\frac{1}{3}$ is $\frac{2}{6}$	$\frac{2}{6}$ are $\frac{1}{3}$
$\frac{2}{3}$ are twice $\frac{2}{6}$ or $\frac{4}{6}$	$\frac{4}{6}$ are twice $\frac{2}{6}$ or $\frac{2}{3}$
$\frac{3}{3}$ are 3 times $\frac{2}{6}$ or $\frac{6}{6}$	$\frac{6}{6}$ are 3 times $\frac{2}{6}$ or $\frac{3}{3}$
$\frac{4}{3}$ are $4 \times \frac{2}{6}$ or $\frac{8}{6}$	$\frac{8}{6}$ are 4 times $\frac{2}{6}$ or $\frac{4}{3}$
&c.	&c.

And so on up to the

Tenth Square,

in which halves and tenths are reduced to the same denominator, and conversely.

1 is $\frac{20}{20}$	$\frac{20}{20}$ are 1
$\frac{1}{2}$ is $\frac{10}{20}$	$\frac{10}{20}$ are $\frac{1}{2}$
$\frac{2}{2}$ are twice $\frac{10}{20}$ or $\frac{20}{20}$	$\frac{20}{20}$ are twice $\frac{10}{20}$ or $\frac{2}{2}$
$\frac{3}{2}$ are 3 times $\frac{10}{20}$ or $\frac{30}{20}$	$\frac{30}{20}$ are 3 times $\frac{10}{20}$ or $\frac{3}{2}$
$\frac{4}{2}$ are $4 \times \frac{10}{20}$ or $\frac{40}{20}$	$\frac{40}{20}$ are $4 \times \frac{10}{20}$ or $\frac{4}{2}$
&c.	&c.

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1 is $\frac{20}{20}$	$\frac{20}{20}$ are 1
$\frac{1}{10}$ is $\frac{2}{20}$	$\frac{2}{20}$ are $\frac{1}{10}$
$\frac{2}{10}$ are twice $\frac{2}{20}$ or $\frac{4}{20}$	$\frac{4}{20}$ are twice $\frac{2}{20}$ or $\frac{2}{10}$
$\frac{3}{10}$ are 3 times $\frac{2}{20}$ or $\frac{6}{20}$	$\frac{6}{20}$ are 3 times $\frac{2}{20}$ or $\frac{3}{10}$
&c.	&c.

Third Line of Squares.

Second Square,

in which thirds and halves are reduced to the same denominator, viz. to sixths, and conversely.

1 is $\frac{6}{6}$	$\frac{6}{6}$ are 1
$\frac{1}{3}$ is $\frac{2}{6}$	$\frac{2}{6}$ are $\frac{1}{3}$
$\frac{2}{3}$ are twice $\frac{2}{6}$ or $\frac{4}{6}$	$\frac{4}{6}$ are twice $\frac{2}{6}$ or $\frac{2}{3}$
$\frac{3}{3}$ are 3 times $\frac{2}{6}$ or $\frac{6}{6}$	$\frac{6}{6}$ are 3 times $\frac{2}{6}$ or $\frac{3}{3}$
&c.	&c.

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1 is $\frac{6}{6}$	$\frac{6}{6}$ are 1
$\frac{1}{2}$ is $\frac{3}{6}$	$\frac{3}{6}$ are $\frac{1}{2}$
$\frac{2}{2}$ are twice $\frac{3}{6}$ or $\frac{6}{6}$	$\frac{6}{6}$ are twice $\frac{3}{6}$ or $\frac{2}{2}$
$\frac{3}{2}$ are 3 times $\frac{3}{6}$ or $\frac{9}{6}$	$\frac{9}{6}$ are 3 times $\frac{3}{6}$ or $\frac{3}{2}$
&c.	&c.

Third Square,

in which thirds are reduced to ninths, and conversely.

1 is $\frac{9}{9}$	$\frac{9}{9}$ are 1
$\frac{1}{3}$ is $\frac{3}{9}$	$\frac{3}{9}$ are $\frac{1}{3}$
$\frac{2}{3}$ are twice $\frac{3}{9}$, or $\frac{6}{9}$	$\frac{6}{9}$ are twice $\frac{3}{9}$, or $\frac{2}{3}$
$\frac{3}{3}$ are 3 times $\frac{3}{9}$, or $\frac{9}{9}$	$\frac{9}{9}$ are 3 times $\frac{3}{9}$, or $\frac{3}{3}$
$\frac{4}{3}$ are 4 times $\frac{3}{9}$, or $\frac{12}{9}$	$\frac{12}{9}$ are 4 times $\frac{3}{9}$, or $\frac{4}{3}$
&c.	&c.

Fourth Square,

in which thirds and fourths are reduced to twelfths, and conversely.

1 is $\frac{12}{12}$	$\frac{12}{12}$ are 1
$\frac{1}{3}$ is $\frac{4}{12}$	$\frac{4}{12}$ are $\frac{1}{3}$
$\frac{2}{3}$ are twice $\frac{4}{12}$, or $\frac{8}{12}$	$\frac{8}{12}$ are twice $\frac{4}{12}$, or $\frac{2}{3}$
$\frac{3}{3}$ are 3 times $\frac{4}{12}$, or $\frac{12}{12}$	$\frac{12}{12}$ are 3 times $\frac{4}{12}$, or $\frac{3}{3}$
$\frac{4}{3}$ are $4 \times \frac{4}{12}$, or $\frac{16}{12}$	$\frac{16}{12}$ are $4 \times \frac{4}{12}$, or $\frac{4}{3}$
&c.	&c.

1 is $\frac{12}{12}$	$\frac{12}{12}$ are 1
$\frac{1}{4}$ are $\frac{3}{12}$	$\frac{3}{12}$ are $\frac{1}{4}$
$\frac{2}{4}$ are twice $\frac{3}{12}$, or $\frac{6}{12}$	$\frac{6}{12}$ are twice $\frac{3}{12}$, or $\frac{2}{4}$
$\frac{3}{4}$ are 3 times $\frac{3}{12}$, or $\frac{9}{12}$	$\frac{9}{12}$ are 3 times $\frac{3}{12}$, or $\frac{3}{4}$
&c.	&c.

And so on with the other squares, concluding with the

Tenth Square,

in which thirds and tenths are reduced to the same denominator, viz., thirtieths, and conversely.

1 is $\frac{30}{30}$	$\frac{30}{30}$ are 1
$\frac{1}{3}$ is $\frac{10}{30}$	$\frac{10}{30}$ are $\frac{1}{3}$
$\frac{2}{3}$ are twice $\frac{10}{30}$, or $\frac{20}{30}$	$\frac{20}{30}$ are twice $\frac{10}{30}$, or $\frac{2}{3}$
$\frac{3}{3}$ are 3 times $\frac{10}{30}$, or $\frac{30}{30}$	$\frac{30}{30}$ are 3 times $\frac{10}{30}$, or $\frac{3}{3}$
&c.	&c.

1 is $\frac{30}{30}$	$\frac{30}{30}$ are 1
$\frac{1}{10}$ is $\frac{3}{30}$	$\frac{3}{30}$ are $\frac{1}{10}$
$\frac{2}{10}$ are twice $\frac{3}{30}$, or $\frac{6}{30}$	$\frac{6}{30}$ are twice $\frac{3}{30}$, or $\frac{2}{10}$
$\frac{3}{10}$ are 3 times $\frac{3}{30}$, or $\frac{9}{30}$	$\frac{9}{30}$ are 3 times $\frac{3}{30}$, or $\frac{3}{10}$
&c.	&c.

This exercise proves that *if the numerator and denominator of a fraction be multiplied by the same number, the fraction is not altered in value.* It will also be seen that as by multiplying the *denominator* of a fraction by 2, 3, 4, &c., we reduce the fraction to one-half, one-third, one-fourth, &c., of its first value; so on the other hand, by multiplying the *numerator* of the fraction by 2, 3, 4, &c., we increase the value of the fraction as much as we decreased it in the former case. These principles are rendered very evident in the above exercises.

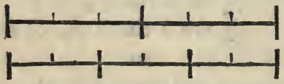
In like manner, it is obvious, that the value of a fraction is not changed by dividing the numerator and denominator by the same number.

A fraction is expressed in its least terms, when the numerator and denominator have no common divisor.

Two fractions are, in general, brought to a *common*, that is, to the *same* denominator, by multiplying the numerator and denominator of each by the denominator of the other.

In the foregoing exercise, fractions have been considered as a part of a unit of a surface; but it may not be inappropriate to exhibit the same results, when the unit is assumed to be a line or a solid. Let A and B represent two rods, or two lineal units, one foot each; A is divided into 2 equal parts, and B into 3.

A
B



In order to reduce the halves thus formed on the first line, and the thirds, on the second line, to a common measuring fraction, we must divide each half into 3 equal parts, and each third into 2 equal parts; for by so doing, both rods will be divided into six equal parts, or sixths. Hence we have, on the rod A, as in the above table, $\frac{1}{2}=\frac{3}{6}$; $\frac{2}{2}=\frac{6}{6}$; &c.; and on the rod B, $\frac{1}{3}=\frac{2}{6}$; $\frac{2}{3}=\frac{4}{6}$; $\frac{3}{3}=\frac{6}{6}$. By means of these rods the formation of compound fractions may also be shown; for example, on the rod B, we see that the half of $\frac{1}{3}$ is $\frac{1}{6}$, and so on.

Again, if an apple be divided into 3 equal parts, each part would be called a third; and if each third be then cut into 2 equal parts, the whole apple would be divided into 6 equal parts, and therefore each part would be called a sixth, two of such parts would be two sixths, which are the same as one-third.

Let it be required to show the sum of one-third and one-half. By the above reasoning, one-third is the same as two-sixths; and proceeding in the same way, I show that one-half of an apple contains three-sixths, and consequently the sum of the third and the half will be the addition of two sixths and three-sixths, which are equal to five-sixths.

Other fractional operations may, in a similar manner, be exhibited. But it is obvious that such processes are more conveniently performed with the assistance of the Table of Compound Fractions.

Questions on the Fifth Exercise.

Second Line.

Second Square.

i. Reduce $\frac{1}{2}$ and $\frac{1}{4}$ to the same denominator.....*Ans.* $\frac{2}{4}$ and $\frac{1}{4}$; because $1 = \frac{4}{4}$, and $\frac{1}{2} =$ the half of $\frac{4}{4}$, that is, $\frac{2}{4}$.

ii. Reduce $\frac{3}{2}$ to fourths.....*Ans.* $\frac{6}{4}$.

iii. Reduce $\frac{4}{8}$ to halves.....*Ans.* $\frac{1}{2}$.

iv. How would you cut a half of a cheese into eighths?

Third Square.

i. In $\frac{2}{3}$ how many sixths?....*Ans.* $\frac{4}{6}$; because $1 = \frac{6}{6}$, and $\frac{1}{3} =$ the third of $\frac{6}{6}$, that is $\frac{2}{6}$; therefore $\frac{2}{3} =$ twice $\frac{2}{6}$ or $\frac{4}{6}$.

ii. Bring $\frac{1}{3}$ and $\frac{1}{2}$ to the same denominator.....*Ans.* $\frac{2}{6}$ and $\frac{3}{6}$.

iii. Reduce $\frac{8}{6}$ to its least terms.....*Ans.* $\frac{4}{3}$.

iv. On which square do you find $\frac{2}{3}$ and $\frac{3}{2}$ brought to the same denominator? *Ans.* On the third square of the second line, where halves and thirds are brought to sixths.

Fourth Square.

i. How many eighths would there be in the half of this rod?....*Ans.* $\frac{4}{8}$.

ii. What are the fractions which this square reduces to the same denominator?....*Ans.* Halves and fourths, which are reduced to eighths.

iii. Reduce $\frac{6}{8}$ to its least term.....*Ans.* $\frac{3}{4}$.

Proof. $\frac{1}{4}$ contains $\frac{2}{8}$, $\frac{2}{4}$ contain $\frac{4}{8}$, and $\frac{3}{4}$ contain $\frac{6}{8}$.

Fifth Square.

i. Reduce $\frac{4}{10}$ to its least terms.....*Ans.* $\frac{2}{5}$.

ii. Bring $\frac{1}{2}$ and $\frac{3}{5}$ to a common denominator.....*Ans.* $\frac{5}{10}$ and $\frac{6}{10}$.

iii. To what fraction must I bring $\frac{3}{2}$ and $\frac{4}{5}$, so that I may be able to add them?....*Ans.* Tenths, as shown on the fifth square of the second line.

And so on to the sixth, seventh, eighth, ninth, and tenth squares.

Third Line.

Second Square.

i. What fractions are reduced by this square to the same denominator?....*Ans.* Halves and thirds.

ii. In one half how many sixths?....*Ans.* Three sixths; because $1 = \frac{6}{6}$; and therefore $\frac{1}{2} =$ the half of $\frac{6}{6}$, that is $\frac{3}{6}$.

iii. Reduce $\frac{1}{2}$ and $\frac{2}{3}$ to fractions having a common denominator.....*Ans.* $\frac{3}{6}$ and $\frac{4}{6}$.

Proof. This operation must be performed on a square which contains halves and thirds, namely, the second square of the third line; where we observe that $\frac{1}{2}$ contains $\frac{3}{6}$, and $\frac{2}{3}$ contain $\frac{4}{6}$.

Third Square.

i. How many ninths are there in $\frac{1}{3}$?....*Ans.* $\frac{3}{9}$; because 1 contains $\frac{9}{9}$; and therefore $\frac{1}{3}$ contains the third of $\frac{9}{9}$, that is, $\frac{3}{9}$.

ii. What is the common denominator to which fractions are reduced by this square?....*Ans.* Ninths.

iii. Reduce $\frac{12}{9}$ to thirds.....*Ans.* $\frac{4}{3}$.

Fourth Square.

i. Reduce $\frac{3}{4}$ and $\frac{2}{3}$ to fractions having the same denominator.....*Ans.* $\frac{9}{12}$ and $\frac{8}{12}$.

ii. In $\frac{7}{3}$ how many twelfths ?....*Ans.* $\frac{28}{12}$.

iii. Reduce $\frac{16}{12}$ to thirds.....*Ans.* $\frac{4}{3}$.

iv. Reduce $\frac{9}{12}$ to fourths.....*Ans.* $\frac{3}{4}$.

v. How many twelfths can I take out of $\frac{2}{3}$?....*Ans.* 8 twelfths.

Fifth Square.

i. What fractions are reduced to the same denominator by this square ?....*Ans.* Thirds and fifths.

ii. In $\frac{5}{15}$ how many thirds ?....*Ans.* $\frac{1}{3}$.

iii. Reduce $\frac{4}{3}$ and $\frac{2}{5}$ to the same denominator.....*Ans.* $\frac{20}{15}$ and $\frac{6}{15}$.

And so on to the sixth, seventh, eighth, ninth, and tenth squares.

Fourth Line.

Second Square.

i. What fractions are reduced by this square to the same denominator ?....*Ans.* Halves and fourths.

ii. In $\frac{2}{4}$ how many eighths ?....*Ans.* $\frac{4}{8}$.

iii. Reduce $\frac{1}{2}$ and $\frac{3}{4}$ to eighths.....*Ans.* $\frac{4}{8}$ and $\frac{6}{8}$.

Third Square.

i. In $\frac{3}{4}$ how many twelfths?....*Ans.* $\frac{9}{12}$; because 1 contains $\frac{12}{12}$, and $\frac{1}{4}$ contains the fourth of $\frac{12}{12}$, that is $\frac{3}{12}$; therefore $\frac{3}{4} =$ 3 times $\frac{3}{12}$, or $\frac{9}{12}$.

ii. What is the common denominator to which fractions are reduced by this square ?....*Ans.* Twelfths.

iii. Reduce $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ to equivalent fractions having like denominators.....*Ans.* $\frac{6}{12}$, $\frac{4}{12}$, and $\frac{3}{12}$.

iv. Reduce $\frac{4}{12}$ to its lowest terms, and tell me on which line and square it is done.....*Ans.* $\frac{4}{12}$ are $\frac{1}{3}$, and it is done on the third square of the fourth line.

v. How would you bring the third of an orange, and the fourth of an orange, to the same parts?....*Ans.* By dividing the third into four equal parts, which would give $\frac{4}{12}$; and then dividing the fourth into three equal parts, which would give $\frac{3}{12}$.

The additions and subtractions performed in the third exercise, were restricted to fractions having the same denominator; but as the pupil is now enabled to reduce any two or more fractions to that state, it will be desirable to terminate this exercise with a few questions in the

Addition and Subtraction of Fractions having different Denominators.

In proposing these questions the teacher is recommended to have the particular square referred to, drawn out upon an enlarged scale and placed before the Class.

On the Board of Compound Fractions.

Second Line of Squares.

Second Square.

i. Add $\frac{1}{2}$ and $\frac{1}{4}$ together.....*Ans.* $\frac{3}{4}$, because $\frac{1}{2} = \frac{2}{4}$; therefore $\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$.

ii. Add $\frac{3}{2}$ and $\frac{3}{4}$ together.....*Ans.* $\frac{9}{4} = 2\frac{1}{4}$, because $\frac{3}{2} = \frac{6}{4}$; and $\frac{6}{4} + \frac{3}{4} = \frac{9}{4}$, or $2\frac{1}{4}$.

iii. What is the difference between $2\frac{1}{4}$ and $1\frac{1}{2}$?....*Ans.* $\frac{3}{4}$.

Third Square.

i. What is the sum of $\frac{1}{2}$ and $\frac{1}{3}$?....*Ans.* $\frac{5}{6}$.

ii. What is the sum of $\frac{1}{2}$ and $\frac{2}{3}$?....*Ans.* $\frac{7}{6} = 1\frac{1}{6}$.

iii. What is the sum of $1\frac{1}{2}$ and $1\frac{1}{3}$?....*Ans.* $\frac{17}{6} = 2\frac{5}{6}$.

iv. What is the difference between $\frac{2}{3}$ and $\frac{1}{2}$?....*Ans.* $\frac{1}{6}$; because $\frac{2}{3}$ are $\frac{4}{6}$; and $\frac{1}{2}$ are $\frac{3}{6}$; therefore $\frac{4}{6}$ less by $\frac{3}{6} = \frac{1}{6}$.

v. What is the sum of $\frac{1}{3}$ and $\frac{1}{6}$?....*Ans.* $\frac{3}{6} = \frac{1}{2}$.

vi. Two-thirds of a cheese, and the one-sixth of the same cheese, make up what part of the whole cheese?....*Ans.* $\frac{5}{6}$.

Fifth Square.

i. What is the sum of $\frac{1}{2}$ and $\frac{1}{5}$?....*Ans.* $\frac{7}{10}$; because $\frac{1}{2} = \frac{5}{10}$; and $\frac{1}{5} = \frac{2}{10}$; therefore $\frac{5}{10} + \frac{2}{10} = \frac{7}{10}$.

ii. What is the sum of $1\frac{1}{2}$ and $\frac{3}{5}$?....*Ans.* $\frac{21}{10} = 2\frac{1}{10}$; because $1\frac{1}{2} = \frac{3}{2} = \frac{15}{10}$; $\frac{3}{5} = \frac{6}{10}$; therefore $\frac{15}{10} + \frac{6}{10} = \frac{21}{10} = 2\frac{1}{10}$.

iii. What is the difference between $\frac{4}{5}$ and $\frac{1}{2}$?....*Ans.* $\frac{3}{10}$ because $\frac{4}{5} = \frac{8}{10}$; and $\frac{1}{2} = \frac{5}{10}$; therefore $\frac{8}{10} - \frac{5}{10} = \frac{3}{10}$.

iv. What is the sum of $\frac{1}{2}$ and $\frac{1}{10}$?....*Ans.* $\frac{6}{10} = \frac{3}{5}$.

And so on to every square in this line.

Third Line of Squares.

Third Square.

i. What is the sum of $\frac{1}{3}$ and $\frac{1}{9}$?....*Ans.* $\frac{4}{9}$.

ii. What is the sum of $\frac{2}{3}$ and $\frac{2}{9}$?....*Ans.* $\frac{8}{9}$.

iii. What is the sum of $1\frac{1}{3}$ and $1\frac{1}{9}$?....*Ans.* $2\frac{4}{9}$.

iv. What is the difference between $\frac{2}{3}$ and $\frac{2}{9}$?....*Ans.* $\frac{4}{9}$; because $\frac{2}{3} = \frac{6}{9}$; and therefore $\frac{6}{9} - \frac{2}{9} = \frac{4}{9}$?

Fourth Square.

i. What is the sum of $\frac{1}{3}$ and $\frac{1}{4}$?....*Ans.* $\frac{7}{12}$.

ii. What is the sum of $\frac{1}{3}$ and $\frac{3}{4}$?....*Ans.* $\frac{13}{12} = 1\frac{1}{12}$.

iii. What is the sum of $\frac{2}{3}$ and $\frac{1}{4}$?....*Ans.* $\frac{11}{12}$.

iv. What is the difference between $2\frac{1}{3}$ and $1\frac{1}{4}$?....*Ans.* $\frac{13}{12} = 1\frac{1}{12}$; because $2\frac{1}{3} = \frac{7}{3} = \frac{28}{12}$; and $1\frac{1}{4} = \frac{5}{4} = \frac{15}{12}$; therefore $\frac{28}{12} - \frac{15}{12} = \frac{13}{12} = 1\frac{1}{12}$.

And so on to any other square.

Fourth Line of Squares.

Second Square.

i. What is the sum of $\frac{1}{2}$ and $\frac{1}{8}$?....*Ans.* $\frac{5}{8}$.

ii. What is the sum of $\frac{1}{2}$ and $\frac{3}{8}$?....*Ans.* $\frac{7}{8}$.

iii. What is the sum and difference of $\frac{3}{2}$ and $\frac{5}{8}$?....*Ans.* $\frac{17}{8}$ and $\frac{7}{8}$.

Fourth Square.

i. What is the sum of $\frac{1}{4}$ and $\frac{1}{16}$?....*Ans.* $\frac{5}{16}$.

ii. What is the difference between $\frac{3}{4}$ and $\frac{1}{16}$?....*Ans.* $\frac{11}{16}$.

iii. A post is $\frac{1}{4}$ in the mud, and $\frac{3}{16}$ in the water, how much does it stand above the water?....*Ans.* $\frac{9}{16}$.

Fifth Square.

i. What is the sum of $\frac{1}{4}$ and $\frac{4}{5}$?....*Ans.* $1\frac{1}{20}$.

ii. What must I take away from $\frac{7}{20}$ to leave $\frac{1}{5}$?....*Ans.* $\frac{3}{20}$.

&c., &c., &c.

Fifth Line of Squares.

Third Square.

i. What is the sum of $\frac{1}{3}$ and $\frac{1}{5}$?....*Ans.* $\frac{8}{15}$.

ii. What is the sum of $\frac{1}{3}$ and $\frac{1}{15}$?....*Ans.* $\frac{6}{15} = \frac{2}{5}$.

iii. What is the sum of $\frac{1}{5}$ and $\frac{1}{15}$?....*Ans.* $\frac{4}{15}$.

iv. What must be added to $\frac{3}{5}$ to make $\frac{13}{15}$?....*Ans.* $\frac{4}{15}$.

Fourth Square.

i. What is the sum of $\frac{1}{4}$ and $\frac{1}{20}$?....*Ans.* $\frac{6}{20} = \frac{3}{10}$.

ii. What is the sum of $\frac{1}{5}$ and $\frac{1}{20}$?....*Ans.* $\frac{5}{20} = \frac{1}{4}$.

iii. What is the sum of $\frac{1}{4}$ and $\frac{1}{5}$?....*Ans.* $\frac{9}{20}$.

iv. What is the sum of $\frac{3}{4}$ and $\frac{2}{5}$?....*Ans.* $\frac{23}{20} = 1\frac{3}{20}$.

&c., &c., &c.

Fifth Square.

i. What is the difference between $\frac{1}{5}$ and $\frac{1}{25}$?....*Ans.* $\frac{4}{25}$.

ii. What is the difference between $\frac{3}{5}$ and $\frac{1}{25}$?....*Ans.* $\frac{14}{25}$.

&c., &c., &c.

Sixth Square.

i. What is the sum of $\frac{1}{6}$ and $\frac{1}{30}$?....*Ans.* $\frac{6}{30} = \frac{1}{5}$.

ii. What is the sum of $\frac{1}{5}$ and $\frac{1}{30}$?....*Ans.* $\frac{7}{30}$.

iii. What is the sum of $\frac{1}{6}$ and $\frac{1}{5}$?....*Ans.* $\frac{11}{30}$.

&c., &c., &c.

And so on to other squares in this line.

In like manner questions on the squares contained in the sixth, seventh, eighth, &c. lines may be given.

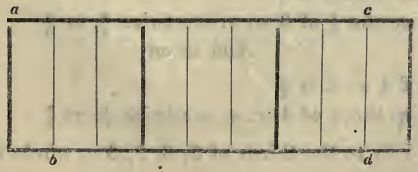
SIXTH EXERCISE.

Multiplication of Fractions.

THE varieties of multiplication by fractions, to which reference was made in the fourth exercise, may now be studied. For the sake of convenience in teaching, this exercise is divided into three cases. 1st. Multiplication of a whole number by a fraction. 2nd. Multiplication of a fraction by a fraction when the numerator of each fraction is unity. 3rd. Multiplication of a fraction by a fraction, when the numerators are whole numbers. For the illustration of the first case, the Board of Simple Fractions is required; for the second case the Board of Compound Fractions; but for the third case both boards will be required. The proper board to be used will be stated before each table.

Case I. *Multiplication of whole numbers by a fraction, or the fraction of a whole number.*

To demonstrate the operations performed with the assistance of this table, it will be found convenient, in certain cases, to draw a few squares, according to the form given in the diagram below, where the space, *a*, *b*, *c*, *d*, for example, represents "4 times the $\frac{1}{3}$ of 2 whole numbers, or 4 times $\frac{2}{3}$, that is, $\frac{8}{3}$."



On the Board of Simple Fractions.

Second Line, or Line of Halves.

$\frac{1}{2}$ of 1 whole number is $\frac{1}{2}$
twice the $\frac{1}{2}$ of 1 w. n. are $\frac{2}{2}$
3 times the $\frac{1}{2}$ of 1 w. n. are $\frac{3}{2}$
And so on.

$\frac{1}{2}$ of 2 w. n. is $\frac{2}{2}$
twice the $\frac{1}{2}$ of 2 w. n. are twice $\frac{2}{2}$, or $\frac{4}{2}$
3 times the $\frac{1}{2}$ of 2 w. n. are 3 times $\frac{2}{2}$, or $\frac{6}{2}$
And so on.

$\frac{1}{2}$ of 3 w. n. is $\frac{3}{2}$
twice the $\frac{1}{2}$ of 3 w. n. are twice $\frac{3}{2}$, or $\frac{6}{2}$
3 times the $\frac{1}{2}$ of 3 w. n. are 3 times $\frac{3}{2}$, or $\frac{9}{2}$
4 times the $\frac{1}{2}$ of 3 w. n. are 4 times $\frac{3}{2}$, or $\frac{12}{2}$
And so on.

The teacher then goes on with the halves of 4, 5, 6, &c., whole numbers.

Third Line, or Line of Thirds.

$\frac{1}{3}$ of 1 w. n. is $\frac{1}{3}$
twice the $\frac{1}{3}$ of 1 w. n. are $\frac{2}{3}$
3 times the $\frac{1}{3}$ of 1 w. n. are $\frac{3}{3}$
4 times the $\frac{1}{3}$ of 1 w. n. are $\frac{4}{3}$
And so on.

$\frac{1}{3}$ of 2 w. n. is $\frac{2}{3}$
twice the $\frac{1}{3}$ of 2 w. n. are twice $\frac{2}{3}$, or $\frac{4}{3}$
3 times the $\frac{1}{3}$ of 2 w. n. are 3 times $\frac{2}{3}$, or $\frac{6}{3}$
4 times the $\frac{1}{3}$ of 2 w. n. are 4 times $\frac{2}{3}$, or $\frac{8}{3}$
And so on.

$\frac{1}{3}$ of 3 w. n. is $\frac{3}{3}$
twice the $\frac{1}{3}$ of 3 w. n. are twice $\frac{3}{3}$, or $\frac{6}{3}$
And so on.

$\frac{1}{3}$ of 4 w. n. is $\frac{4}{3}$
twice the $\frac{1}{3}$ of 4 w. n. are twice $\frac{4}{3}$, or $\frac{8}{3}$

And so on to the thirds of 5, 6, 7, &c., whole numbers.

Fourth Line, or Line of Fourths.

$\frac{1}{4}$ of 1 w. n. is $\frac{1}{4}$
twice the $\frac{1}{4}$ of 1 w. n. are $\frac{2}{4}$
3 times the $\frac{1}{4}$ of 1 w. n. are $\frac{3}{4}$
And so on.

$\frac{1}{4}$ of 2 w. n. is $\frac{2}{4}$
twice the $\frac{1}{4}$ of 2 w. n. are twice $\frac{2}{4}$, or $\frac{4}{4}$
3 times the $\frac{1}{4}$ of 2 w. n. are 3 times $\frac{2}{4}$, or $\frac{6}{4}$
4 times the $\frac{1}{4}$ of 2 w. n. are 4 times $\frac{2}{4}$, or $\frac{8}{4}$
And so on.

$\frac{1}{4}$ of 3 w. n. is $\frac{3}{4}$
twice the $\frac{1}{4}$ of 3 w. n. are twice $\frac{3}{4}$, or $\frac{6}{4}$
3 times the $\frac{1}{4}$ of 3 w. n. are 3 times $\frac{3}{4}$, or $\frac{9}{4}$
4 times the $\frac{1}{4}$ of 3 w. n. are 4 times $\frac{3}{4}$, or $\frac{12}{4}$
5 times the $\frac{1}{4}$ of 3 w. n. are 5 times $\frac{3}{4}$, or $\frac{15}{4}$

And so on to the fractional parts of 4, 5, 6, &c., whole numbers.

Fifth Line, or Line of Fifths.

$\frac{1}{5}$ of 1 w. n. is $\frac{1}{5}$
twice the $\frac{1}{5}$ of 1 w. n. are $\frac{2}{5}$
3 times the $\frac{1}{5}$ of 1 w. n. are $\frac{3}{5}$
And so on.

$\frac{1}{5}$ of 2 w. n. is $\frac{2}{5}$
twice the $\frac{1}{5}$ of 2 w. n. are twice $\frac{2}{5}$, or $\frac{4}{5}$
3 times the $\frac{1}{5}$ of 2 w. n. are 3 times $\frac{2}{5}$, or $\frac{6}{5}$
And so on.

$\frac{1}{5}$ of 3 w. n. is $\frac{3}{5}$
twice the $\frac{1}{5}$ of 3 w. n. are twice $\frac{3}{5}$, or $\frac{6}{5}$
3 times the $\frac{1}{5}$ of 3 w. n. are 3 times $\frac{3}{5}$, or $\frac{9}{5}$
And so on.

$\frac{1}{5}$ of 4 w. n. is $\frac{4}{5}$
twice the $\frac{1}{5}$ of 4 w. n. are twice $\frac{4}{5}$, or $\frac{8}{5}$
3 times the $\frac{1}{5}$ of 4 w. n. are 3 times $\frac{4}{5}$, or $\frac{12}{5}$

And so on to the fractional parts of 5, 6, 7, 8, &c., whole numbers.

Sixth Line, or Line of Sixths.

$\frac{1}{6}$ of 1 w. n. is $\frac{1}{6}$
twice the $\frac{1}{6}$ of 1 w. n. are $\frac{2}{6}$
3 times the $\frac{1}{6}$ of 1 w. n. are $\frac{3}{6}$
And so on.

$\frac{1}{6}$ of 2 w. n. is $\frac{2}{6}$
twice the $\frac{1}{6}$ of 2 w. n. are twice $\frac{2}{6}$, or $\frac{4}{6}$
3 times the $\frac{1}{6}$ of 2 w. n. are 3 times $\frac{2}{6}$, or $\frac{6}{6}$
And so on.

$\frac{1}{6}$ of 3 w. n. is $\frac{3}{6}$
twice the $\frac{1}{6}$ of 3 w. n. are twice $\frac{3}{6}$, or $\frac{6}{6}$
3 times the $\frac{1}{6}$ of 3 w. n. are 3 times $\frac{3}{6}$, or $\frac{9}{6}$

And so on to the lines of sevenths, eighths, ninths, &c., &c.

Questions on Case I.

Line of Halves.

i. What is the half of 3?....*Ans.* $\frac{3}{2}$; because the $\frac{1}{2}$ of $1 = \frac{1}{2}$, and therefore the $\frac{1}{2}$ of $3 = 3$ times $\frac{1}{2}$, that is $\frac{3}{2}$.

ii. What is 3 times the $\frac{1}{2}$ of 5?....*Ans.* $\frac{15}{2} = 7\frac{1}{2}$; because the $\frac{1}{2}$ of $5 = \frac{5}{2}$, and therefore $3 \times \frac{1}{2}$ of $5 = 3 \times \frac{5}{2}$ or $\frac{15}{2}$.

&c. &c.

Line of Thirds.

i. What is 7 times the $\frac{1}{3}$ of 5 whole numbers?....*Ans.* $\frac{35}{3}$ or $11\frac{2}{3}$; because the $\frac{1}{3}$ of 5 whole numbers is $\frac{5}{3}$, and 7 times the $\frac{1}{3}$ of 5 whole numbers are 7 times $\frac{5}{3}$ or $\frac{35}{3}$.

ii. What is $\frac{2}{3}$ of 4 feet?....*Ans.* 2 feet and $\frac{2}{3}$; because the third of 4 feet is $\frac{4}{3}$ of 1 foot, and twice the $\frac{1}{3}$ of 4 feet are twice $\frac{4}{3}$ or $\frac{8}{3}$ of a foot, which are 2 feet and $\frac{2}{3}$.

Line of Fourths.

i. What is the fourth of 3?....*Ans.* $\frac{3}{4}$; because the fourth of $1 = \frac{1}{4}$, and therefore $\frac{1}{4}$ of $3 = 3$ times $\frac{1}{4}$, that is $\frac{3}{4}$.

ii. What is 3 times the $\frac{1}{4}$ of 7?....*Ans.* $\frac{21}{4}$ or $5\frac{1}{4}$; because

the $\frac{1}{4}$ of 7 = $\frac{7}{4}$, and therefore 3 times the $\frac{1}{4}$ of 7 = 3 times $\frac{7}{4}$ = $\frac{21}{4}$ or $5\frac{1}{4}$.

iii. What does it mean to multiply 7 by $\frac{3}{4}$?....*Ans.* That the fourth of 7 is to be taken 3 times.

Line of Fifths.

i. How many fifths are there in the fifth of 9 ?....*Ans.* $\frac{9}{5}$; because $\frac{1}{5}$ of 1 = $\frac{1}{5}$: and therefore the $\frac{1}{5}$ of 9 = 9 times $\frac{1}{5}$ or $\frac{9}{5}$.

ii. What is the $\frac{1}{5}$ of 11 ?....*Ans.* $\frac{11}{5}$ = $2\frac{1}{5}$; because $\frac{1}{5}$ of 1 = $\frac{1}{5}$ and the $\frac{1}{5}$ of 11 = 11 times $\frac{1}{5}$ or $\frac{11}{5}$ = $2\frac{1}{5}$.

Money, Weights, and Measures.

Line of Halves.

i. What is the $\frac{1}{2}$ of 5*l.*?....*Ans.* 2*l.* 10*s.*; because the $\frac{1}{2}$ of 5*l.* is $\frac{5}{2}$. The $\frac{1}{2}$ of 1*l.* is 10*s.*, and $\frac{5}{2}$*l.* are 5 times 10*s.* or 2*l.* 10*s.*

ii. If 2 yards of cloth cost 3*s.*, what will 7 yards cost?.... *Ans.* 10*s.* 6*d.*; because one yard will cost the $\frac{1}{2}$ of 3*s.*, that is, $\frac{3}{2}$*s.*, and therefore 7 yards will cost 7 times $\frac{3}{2}$ or $\frac{21}{2}$*s.*, which are $10\frac{1}{2}$ shillings = 10*s.* 6*d.*

Line of Thirds.

i. If 3 lbs. cost 7*s.*, what will 11 lbs. cost?....*Ans.* 1*l.* 5*s.* 8*d.*; because 3 lbs. cost 7*s.*, 1 lb. will cost the $\frac{1}{3}$ of 7*s.*, that is, $\frac{7}{3}$ of a shilling; 11 lbs. therefore will cost 11 times $\frac{7}{3}$ or $\frac{77}{3}$*s.*, that is, $25\frac{2}{3}$*s.* = 1*l.* 5*s.* 8*d.*

ii. What is the $\frac{1}{3}$ of 1 lb. ?....*Ans.* $5\frac{1}{3}$ ozs.; because 1 lb. contains 16 oz., and the $\frac{1}{3}$ of 16 is $\frac{16}{3}$ or $5\frac{1}{3}$ oz.

Line of Fourths.

i. If 4 articles cost 9*d.*, what will 5 cost ?....*Ans.* $11\frac{1}{4}$*d.*; because if 4 cost 9*d.*, 1 will cost the $\frac{1}{4}$ of 9*d.* or $\frac{9}{4}$ of a penny; and 5 will cost 5 times $\frac{9}{4}$ or $\frac{45}{4}$, that is, $11\frac{1}{4}$*d.*

ii. What is the $\frac{1}{4}$ of 11*l.* ?....*Ans.* 2*l.* 15*s.*; because the $\frac{1}{4}$ of 11 is $\frac{11}{4}$; the $\frac{1}{4}$ of 1*l.* is 5*s.*, and $\frac{11}{4}$ of 1*l.* will be 11 times 5*s.* or 2*l.* 15*s.*

Miscellaneous Questions on Case I.

i. The fifth of $3l.$ is what part of a pound?....*Ans.* $\frac{3}{5}$.

ii. Twice the fifth of a lb. is what part of an ounce? ....*Ans.* $\frac{32}{5}$.

iii. How many feet are there in $\frac{4}{5}$ of a yard?....*Ans.* $\frac{12}{5}=2\frac{2}{5}$ feet.

iv. What is the $\frac{1}{4}$ of $3l.$?....*Ans.* $15s.$; because the $\frac{1}{4}$ of 3 is $\frac{3}{4}$; the $\frac{1}{4}$ of $1l.$ is $5s.$, and $\frac{3}{4}$ of $1l.$ are 3 times $5s.$ or $15s.$

v. What is the $\frac{1}{3}$ of $2s.$?....*Ans.* $8d.$; because the $\frac{1}{3}$ of 2 is $\frac{2}{3}$, the $\frac{1}{3}$ of $1s.$ is $4d.$, and $\frac{2}{3}$ of $1s.$ are twice $4d.$ or $8d.$

vi. What is 3 times the $\frac{1}{5}$ of 1 lb.?....*Ans.* $9\frac{3}{5}$ ozs.; because in 1 lb. there are 16 oz.; the $\frac{1}{5}$ of 16 is $\frac{16}{5}$, and $\frac{3}{5}$ of 16 are 3 times $\frac{16}{5}$ or $\frac{48}{5}=9\frac{3}{5}$.

vii. What is 7 times the $\frac{1}{9}$ of 11 yards?....*Ans.* $8\frac{5}{9}$ yards; because the $\frac{1}{9}$ of 11 is $\frac{11}{9}$, and 7 times the $\frac{1}{9}$ of 11 are 7 times $\frac{11}{9}$ or $\frac{77}{9}=8\frac{5}{9}$.

Case II. *To multiply a fraction by a fraction, or the fraction of a fraction, when the numerator of each fraction is* 1.

On the Board of Compound Fractions.

Second Line.

The half of $\frac{1}{2}=\frac{1}{4}$
The half of $\frac{1}{3}=\frac{1}{6}$
The half of $\frac{1}{4}=\frac{1}{8}$
And so on up to
The half of $\frac{1}{10}=\frac{1}{20}$

The third of $\frac{1}{2}=\frac{1}{6}$
The fourth of $\frac{1}{2}=\frac{1}{8}$
And so on up to
The tenth of $\frac{1}{2}=\frac{1}{20}$

Third Line.

The third of $\frac{1}{2}=\frac{1}{6}$
The third of $\frac{1}{3}=\frac{1}{9}$
The third of $\frac{1}{4}=\frac{1}{12}$
And so on up to
The third of $\frac{1}{10}=\frac{1}{30}$

The half of $\frac{1}{3}=\frac{1}{6}$
The third of $\frac{1}{3}=\frac{1}{9}$
The fourth of $\frac{1}{3}=\frac{1}{12}$
And so on up to
The tenth of $\frac{1}{3}=\frac{1}{30}$

And so on until we arrive at the

Tenth Line.

The tenth of $\frac{1}{2} = \frac{1}{20}$
The tenth of $\frac{1}{3} = \frac{1}{30}$
The tenth of $\frac{1}{4} = \frac{1}{40}$
The tenth of $\frac{1}{5} = \frac{1}{50}$
And so on up to
The tenth of $\frac{1}{10} = \frac{1}{100}$

The half of $\frac{1}{10} = \frac{1}{20}$
The third of $\frac{1}{10} = \frac{1}{30}$
The fourth of $\frac{1}{10} = \frac{1}{40}$
The fifth of $\frac{1}{10} = \frac{1}{50}$

Questions on Case II.

Second Line.

i. If I divide the line A D into three equal parts, —

what part of the whole line is one of these parts, as A C?.... *Ans.* One-third. Because it is one of the three equal parts into which the line is divided.

Let this third be divided into two equal parts at *n*, what part of $\frac{1}{3}$ will one of them (A *n*) be?....*Ans.* $\frac{1}{2}$ of $\frac{1}{3}$.

But what part of the whole line is A *n*?....*Ans.* One sixth. Because each third contains two equal parts, and the whole line is thus divided into 6 equal parts.

What does this prove?....*Ans.* That the $\frac{1}{2}$ of $\frac{1}{3}$ is $\frac{1}{6}$.

ii. What is the half of a half?....*Ans.* $\frac{1}{4}$; because when a half is divided into two equal parts, one part is the fourth of the whole.

iii. What is meant by the half of $\frac{1}{5}$?....*Ans.* That if the unit be divided into five equal parts, then the half of one of these parts is the $\frac{1}{2}$ of $\frac{1}{5}$, which will be $\frac{1}{10}$.

iv. Where do you prove that the $\frac{1}{2}$ of $\frac{1}{6}$ is $\frac{1}{12}$?....*Ans.* On the sixth square of the second line on the Compound Board, where the unit is divided into six equal parts by horizontal lines, and each $\frac{1}{6}$ thus formed is divided into two equal parts by an upright line, making 12 of these in the whole unit, hence the $\frac{1}{2}$ of $\frac{1}{6}$ is $\frac{1}{12}$.

v. Multiply $\frac{1}{4}$ by $\frac{1}{2}$.....*Ans.* $\frac{1}{8}$. To take $\frac{1}{4}$ one-half times is the same as the $\frac{1}{2}$ of $\frac{1}{4}$, which is $\frac{1}{8}$.

vi. What is $\frac{1}{2}$ of $\frac{1}{7}$?....*Ans.* $\frac{1}{14}$.

vii. What is $\frac{1}{2}$ of $\frac{1}{8}$?....*Ans.* $\frac{1}{16}$.

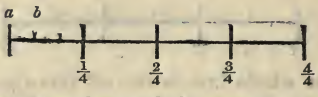
viii. What is $\frac{1}{2}$ of $\frac{1}{10}$?....*Ans.* $\frac{1}{20}$.

Third Line.

i. What is $\frac{1}{3}$ of $\frac{1}{3}$?....*Ans.* $\frac{1}{9}$; because if a third be divided into three equal parts, there would be nine of such parts to make up the whole unit, hence $\frac{1}{3}$ of $\frac{1}{3}=\frac{1}{9}$.

ii. Where do you find the $\frac{1}{3}$ of $\frac{1}{5}$?....*Ans.* On the fifth square of the third line.

iii. Show by the division of a line that the $\frac{1}{3}$ of $\frac{1}{4}$ is $\frac{1}{12}$. ....*Ans.* In the following figure, the part *a b* is the third of $\frac{1}{4}$



and at the same time the twelfth part of the whole line or unit.

iv. Multiply $\frac{1}{7}$ by $\frac{1}{3}$.....*Ans.* $\frac{1}{21}$.

v. Multiply $\frac{1}{9}$ by $\frac{1}{3}$.....*Ans.* $\frac{1}{27}$.

vi. Multiply $\frac{1}{10}$ by $\frac{1}{3}$.....*Ans.* $\frac{1}{30}$.

vii. Show how John can give his friend the third of one-half of an apple.

Fourth Line.

i. Where do you find $\frac{1}{4}$ of $\frac{1}{5}$?....*Ans.* On the fifth square of the fourth line.

ii. What is $\frac{1}{4}$ of $\frac{1}{3}$?....*Ans.* $\frac{1}{12}$.

iii. What is $\frac{1}{4}$ of $\frac{1}{6}$?....*Ans.* $\frac{1}{24}$.

iv. What is $\frac{1}{4}$ of $\frac{1}{7}$?....*Ans.* $\frac{1}{28}$.

v. Multiply $\frac{1}{8}$ by $\frac{1}{4}$. ....*Ans.* $\frac{1}{32}$.

vi. Multiply $\frac{1}{9}$ by $\frac{1}{4}$?....*Ans.* $\frac{1}{36}$.

vii. If an orange be divided into three equal parts, into how many pieces must I cut one of these parts so as to obtain the one-twelfth of the orange?....*Ans.* 4 pieces.

Fifth Line.

i. Where do you find the $\frac{1}{5}$ of $\frac{1}{7}$?....*Ans.* On the seventh square of the fifth line.

ii. Show by the division of an apple that the $\frac{1}{5}$ of $\frac{1}{3}$ is $\frac{1}{15}$. ....*Ans.* First I divide the apple into three equal parts, each of which is $\frac{1}{3}$. I then divide each third into five equal parts, each of which will therefore be the $\frac{1}{5}$ of $\frac{1}{3}$; and the whole apple is now divided into fifteen equal pieces: hence the $\frac{1}{5}$ of $\frac{1}{3} = \frac{1}{15}$.

iii. What is the $\frac{1}{5}$ of $\frac{1}{7}$?....*Ans.* $\frac{1}{35}$.

iv. Show now by a direct process that the $\frac{1}{5}$ of $\frac{1}{2}$ is $\frac{1}{10}$. ....*Ans.* $1 = \frac{10}{10}$, and $\frac{1}{2} =$ the half of $\frac{10}{10}$ or $\frac{5}{10}$; therefore the fifth of $\frac{1}{2}$ is the fifth of $\frac{5}{10} = \frac{1}{10}$.

Similar questions may be given on the remaining lines.

Case III. *When the numerators of the fractions are whole numbers.*

I. *On the Board of Simple Fractions.*

Line of Thirds.

The half of $\frac{2}{3}$ is $\frac{1}{3}$
3 times $\frac{1}{2}$ of $\frac{2}{3}$ are 3 times $\frac{1}{3}$ or $\frac{3}{3}$
*4 times $\frac{1}{2}$ of $\frac{2}{3}$ are 4 times $\frac{1}{3}$ or $\frac{4}{3}$
5 times $\frac{1}{2}$ of $\frac{2}{3}$ are 5 times $\frac{1}{3}$ or $\frac{5}{3}$
&c.

The half of $\frac{4}{3}$ is $\frac{2}{3}$
Twice the $\frac{1}{2}$ of $\frac{4}{3}$ are twice $\frac{2}{3}$ or $\frac{4}{3}$
3 times $\frac{1}{2}$ of $\frac{4}{3}$ are 3 times $\frac{2}{3}$ or $\frac{6}{3}$
4 times $\frac{1}{2}$ of $\frac{4}{3}$ are 4 times $\frac{2}{3}$ or $\frac{8}{3}$
5 times $\frac{1}{2}$ of $\frac{4}{3}$ are 5 times $\frac{2}{3}$ or $\frac{10}{3}$
&c.

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* Read "Four times the half of two-thirds are four times one-third, or four-thirds."

The half of $\frac{6}{3}$ is $\frac{3}{3}$
3 times the $\frac{1}{2}$ of $\frac{6}{3}$ are 3 times $\frac{3}{3}$ or $\frac{9}{3}$
4 times $\frac{1}{2}$ of $\frac{6}{3}$ are 4 times $\frac{3}{3}$ or $\frac{12}{3}$
5 times $\frac{1}{2}$ of $\frac{6}{3}$ are 5 times $\frac{3}{3}$ or $\frac{15}{3}$
6 times $\frac{1}{2}$ of $\frac{6}{3}$ are 6 $\times$ $\frac{3}{3}$ or $\frac{18}{3}$
&c.

The half of $\frac{8}{3}$ is $\frac{4}{3}$
Twice the $\frac{1}{2}$ of $\frac{8}{3}$ are twice $\frac{4}{3}$ or $\frac{8}{3}$
3 times $\frac{1}{2}$ of $\frac{8}{3}$ are 3 times $\frac{4}{3}$ or $\frac{12}{3}$
4 times $\frac{1}{2}$ of $\frac{8}{3}$ are 4 times $\frac{4}{3}$ or $\frac{16}{3}$
5 times $\frac{1}{2}$ of $\frac{8}{3}$ are 5 times $\frac{4}{3}$ or $\frac{20}{3}$

And so on to the processes beginning with

The $\frac{1}{2}$ of $\frac{10}{3}$ is $\frac{5}{3}$

The $\frac{1}{2}$ of $\frac{12}{3}$ is $\frac{6}{3}$

The $\frac{1}{2}$ of $\frac{14}{3}$ is $\frac{7}{3}$, and so on.

The $\frac{1}{4}$ of $\frac{4}{3}$ is $\frac{1}{3}$
Twice $\frac{1}{4}$ of $\frac{4}{3}$ are twice $\frac{1}{3}$ or $\frac{2}{3}$
3 $\times$ $\frac{1}{4}$ of $\frac{4}{3}$ are 3 $\times$ $\frac{1}{3}$ or $\frac{3}{3}$
4 $\times$ $\frac{1}{4}$ of $\frac{4}{3}$ are 4 $\times$ $\frac{1}{3}$ or $\frac{4}{3}$
&c.

The $\frac{1}{4}$ of $\frac{8}{3}$ is $\frac{2}{3}$
Twice the $\frac{1}{4}$ of $\frac{8}{3}$ are twice $\frac{2}{3}$ or $\frac{4}{3}$
3 times the $\frac{1}{4}$ of $\frac{8}{3}$ are 3 times $\frac{2}{3}$ or $\frac{6}{3}$
4 $\times$ $\frac{1}{4}$ of $\frac{8}{3}$ are 4 $\times$ $\frac{2}{3}$ or $\frac{8}{3}$
&c.

The $\frac{1}{4}$ of $\frac{12}{3}$ is $\frac{3}{3}$
Twice $\frac{1}{4}$ of $\frac{12}{3}$ are twice $\frac{3}{3}$ or $\frac{6}{3}$
3 times $\frac{1}{4}$ of $\frac{12}{3}$ are 3 times $\frac{3}{3}$ or $\frac{9}{3}$
4 times $\frac{1}{4}$ of $\frac{12}{3}$ are 4 times $\frac{3}{3}$ or $\frac{12}{3}$
&c.

The $\frac{1}{4}$ of $\frac{16}{3}$ is $\frac{4}{3}$
Twice $\frac{1}{4}$ of $\frac{16}{3}$ are twice $\frac{4}{3}$ or $\frac{8}{3}$
3 times $\frac{1}{4}$ of $\frac{16}{3}$ are 3 times $\frac{4}{3}$ or $\frac{12}{3}$
4 times $\frac{1}{4}$ of $\frac{16}{3}$ are 4 times $\frac{4}{3}$ or $\frac{16}{3}$

And so on to the processes beginning with

The $\frac{1}{4}$ of $\frac{20}{3}$ is $\frac{5}{3}$

The $\frac{1}{4}$ of $\frac{24}{3}$ is $\frac{6}{3}$

The $\frac{1}{4}$ of $\frac{28}{3}$ is $\frac{7}{3}$, and so on.

The $\frac{1}{5}$ of $\frac{5}{3}$ is $\frac{1}{3}$
Twice $\frac{1}{5}$ of $\frac{5}{3}$ are twice $\frac{1}{3}$ or $\frac{2}{3}$
3 times $\frac{1}{5}$ of $\frac{5}{3}$ are 3 times $\frac{1}{3}$ or $\frac{3}{3}$
4 times $\frac{1}{5}$ of $\frac{5}{3}$ are 4 times $\frac{1}{3}$ or $\frac{4}{3}$
&c.

The $\frac{1}{5}$ of $\frac{10}{3}$ is $\frac{2}{3}$
Twice the $\frac{1}{5}$ of $\frac{10}{3}$ are twice $\frac{2}{3}$ or $\frac{4}{3}$
3 times $\frac{1}{5}$ of $\frac{10}{3}$ are 3 times $\frac{2}{3}$ or $\frac{6}{3}$
4 times $\frac{1}{5}$ of $\frac{10}{3}$ are 4 times $\frac{2}{3}$ or $\frac{8}{3}$
&c.

The $\frac{1}{5}$ of $\frac{15}{3}$ is $\frac{3}{3}$
Twice the $\frac{1}{5}$ of $\frac{15}{3}$ are twice $\frac{3}{3}$ or $\frac{6}{3}$
3 times $\frac{1}{5}$ of $\frac{15}{3}$ are 3 times $\frac{3}{3}$ or $\frac{9}{3}$
4 times $\frac{1}{5}$ of $\frac{15}{3}$ are 4 times $\frac{3}{3}$ or $\frac{12}{3}$
&c.

The $\frac{1}{5}$ of $\frac{20}{3}$ is $\frac{4}{3}$
Twice the $\frac{1}{5}$ of $\frac{20}{3}$ are twice $\frac{4}{3}$ or $\frac{8}{3}$
3 times $\frac{1}{5}$ of $\frac{20}{3}$ are 3 times $\frac{4}{3}$ or $\frac{12}{3}$
4 times $\frac{1}{5}$ of $\frac{20}{3}$ are 4 times $\frac{4}{3}$ or $\frac{16}{3}$
&c.

And so on to the processes beginning with

The $\frac{1}{5}$ of $\frac{25}{3}$ is $\frac{5}{3}$

The $\frac{1}{5}$ of $\frac{30}{3}$ is $\frac{6}{3}$

The $\frac{1}{5}$ of $\frac{35}{3}$ is $\frac{7}{3}$

In the same manner may the tables of halves, fourths, fifths, sixths, &c., be constructed.

Questions on Case III.

Line of Halves.

i. What is the $\frac{1}{3}$ of $\frac{9}{2}$?....*Ans.* $\frac{3}{2} = 1\frac{1}{2}$.

ii. What is the $\frac{1}{5}$ of $\frac{25}{2}$?....*Ans.* $\frac{5}{2} = 2\frac{1}{2}$.

iii. What is the $\frac{1}{7}$ of $10\frac{1}{2}$?....*Ans.* $\frac{3}{2} = 1\frac{1}{2}$; because $10\frac{1}{2} = \frac{21}{2}$, and the seventh of $\frac{21}{2} = \frac{3}{2}$ or $1\frac{1}{2}$.

iv. What is the half of $\frac{7}{2}$?....*Ans.* $\frac{7}{4} = 1\frac{3}{4}$; because $\frac{7}{2} = \frac{14}{4}$, and the half of $\frac{14}{4} = \frac{7}{4}$, or $1\frac{3}{4}$.

v. What is $\frac{2}{3}$ of $\frac{9}{2}$?....*Ans.* $\frac{6}{2}$ or 3; because the third of $\frac{9}{2}$ is $\frac{3}{2}$, and therefore twice the third of $\frac{9}{2}$ are twice $\frac{3}{2} = \frac{6}{2}$ or 3.

Line of Thirds.

i. What is the $\frac{1}{4}$ of $\frac{16}{3}$?....*Ans.* $\frac{4}{3}$ or $1\frac{1}{3}$.

ii. What is the $\frac{1}{5}$ of $\frac{10}{3}$?....*Ans.* $\frac{2}{3}$.

iii. What is the $\frac{2}{5}$ of $\frac{25}{3}$? ....*Ans.* $\frac{10}{3} = 3\frac{1}{3}$; because the fifth of $\frac{25}{3}$ is $\frac{5}{3}$; and therefore twice $\frac{1}{5}$ of $\frac{25}{3}$ is twice $\frac{5}{3} = \frac{10}{3}$ or $3\frac{1}{3}$.

iv. What is meant by $\frac{3}{4}$ of $\frac{16}{3}$?....*Ans.* That the fourth of $\frac{16}{3}$ which is $\frac{4}{3}$, is to be taken 3 times.

v. What is the product of $\frac{10}{3}$ by $2\frac{1}{2}$?....*Ans.* $\frac{25}{3} = 8\frac{1}{3}$; because $2\frac{1}{2} = \frac{5}{2}$, and 5 times $\frac{1}{2}$ of $\frac{10}{3}$ are 5 times $\frac{5}{3}$, that is $\frac{25}{3}$.

Line of Fourths.

i. What is $\frac{3}{5}$ of $\frac{10}{4}$?....*Ans.* $\frac{6}{4} = 1\frac{1}{2}$.

ii. What is $\frac{2}{3}$ of $\frac{9}{4}$?....*Ans.* $\frac{6}{4} = 1\frac{1}{2}$.

iii. What is $\frac{3}{7}$ of $\frac{21}{4}$?....*Ans.* $\frac{9}{4} = 2\frac{1}{4}$.

iv. What is $\frac{8}{9}$ of $2\frac{1}{4}$?....*Ans.* $\frac{8}{4} = 2$.

v. Multiply $\frac{5}{4}$ by $\frac{2}{5}$.....*Ans.* $\frac{2}{4} = \frac{1}{2}$.

vi. Multiply $\frac{9}{4}$ by $\frac{2}{9}$.....*Ans.* $\frac{2}{4} = \frac{1}{2}$.

vii. Multiply $3\frac{1}{4}$ by $\frac{2}{13}$.....*Ans.* $\frac{2}{4} = \frac{1}{2}$.

viii. Multiply $6\frac{1}{4}$ by $\frac{3}{5}$.....*Ans.* $\frac{15}{4} = 3\frac{3}{4}$.

Line of Fifths.

i. What is the $\frac{2}{3}$ of $\frac{3}{5}$?....*Ans.* $\frac{2}{5}$.

ii. What is the $\frac{5}{6}$ of $3\frac{3}{5}$?....*Ans.* $\frac{15}{5} = 3$.

iii. Multiply $3\frac{1}{5}$ by $1\frac{1}{4}$.....*Ans.* $\frac{20}{5} = 4$; because $3\frac{1}{5} = \frac{16}{5}$; and $1\frac{1}{4} = \frac{5}{4}$; therefore $5 \times \frac{1}{4}$ of $\frac{16}{5} = 5 \times \frac{4}{5}$ or $\frac{20}{5} = 4$.

&c., &c., &c.

Money, Weights, and Measures.

Line of Halves.

i. $\frac{2}{3}$ of $\frac{9}{2}$ of $1l$. ?....*Ans.* $3l$.; because twice the $\frac{1}{3}$ of $\frac{9}{2}$ are twice $\frac{3}{2}$ or $\frac{6}{2} = 3$.

ii. 5 times the $\frac{1}{7}$ of $\frac{21}{2}$ of $3l$. ?....*Ans.* $\frac{45}{2}l. = 22l.\ 10s.$

Line of Thirds.

i. $\frac{2}{5}$ of $\frac{10}{3}$ of 2 lbs. ?....*Ans.* $2\frac{2}{3}$ lbs.; because $\frac{2}{5}$ of $\frac{10}{3}$ are twice $\frac{2}{3}$ or $\frac{4}{3}$; then $\frac{4}{3}$ of 2 are 4 times $\frac{2}{3}$ or $\frac{8}{3} = 2\frac{2}{3}$.

ii. $\frac{3}{4}$ of $\frac{8}{3}$ of 1 cwt. ?....*Ans.* 2 cwts.

Line of Fourths.

i. $\frac{1}{3}$ of $\frac{3}{4}$ of $1l$. ?....*Ans.* $5s$.; because $\frac{1}{3}$ of $\frac{3}{4}$ is $\frac{1}{4}$, and the $\frac{1}{4}$ of $1l$. is $5s$.

ii. $\frac{3}{5}$ of $\frac{5}{4}$ of $1s$. ?....*Ans.* $\frac{3}{4}$ of $1s$., or $9d$.

Case III., continued.

On the Board of Compound Fractions.

The number of forms in which two fractions may be multiplied together, being almost endless, we shall merely give a specimen arising out of the third square of the fourth line, the second of the fifth, and the fourth of the sixth. In order to extend the table, the teacher will find it convenient to draw on a large scale, the figures annexed to each table after the manner shown at page 92.

Third Square of the Fourth Line.

$\frac{1}{3}$ of $\frac{1}{4} = \frac{1}{12}$	$\frac{1}{4}$ of $\frac{1}{3} = \frac{1}{12}$
*$2 \times \frac{1}{3}$ of $\frac{1}{4} = 2 \times \frac{1}{12}$ or $\frac{2}{12}$	$\frac{1}{4}$ of $\frac{2}{3} = \frac{2}{12}$ or $\frac{1}{6}$
$3 \times \frac{1}{3}$ of $\frac{1}{4} = 3 \times \frac{1}{12}$ or $\frac{3}{12}$	$\frac{1}{4}$ of $\frac{3}{3} = \frac{3}{12}$ or $\frac{1}{4}$
$4 \times \frac{1}{3}$ of $\frac{1}{4} = 4 \times \frac{1}{12}$ or $\frac{4}{12}$	$\frac{1}{4}$ of $\frac{4}{3} = \frac{4}{12}$ or $\frac{1}{3}$
$5 \times \frac{1}{3}$ of $\frac{1}{4} = 5 \times \frac{1}{12}$ or $\frac{5}{12}$	$\frac{1}{4}$ of $\frac{5}{3} = \frac{5}{12}$

$\frac{1}{3}$ of $\frac{2}{4} = \frac{2}{12}$	$\frac{2}{4}$ of $\frac{1}{3} = \frac{2}{12}$ or $\frac{1}{6}$
$2 \times \frac{1}{3}$ of $\frac{2}{4} = \frac{4}{12}$	$\frac{2}{4}$ of $\frac{2}{3} = \frac{4}{12}$ or $\frac{1}{3}$
$3 \times \frac{1}{3}$ of $\frac{2}{4} = \frac{6}{12}$	$\frac{2}{4}$ of $\frac{3}{3} = \frac{6}{12}$ or $\frac{1}{2}$
$4 \times \frac{1}{3}$ of $\frac{2}{4} = \frac{8}{12}$	$\frac{2}{4}$ of $\frac{4}{3} = \frac{8}{12}$ or $\frac{2}{3}$
&c.	&c.

$\frac{1}{3}$ of $\frac{3}{4} = \frac{3}{12}$	$\frac{3}{4}$ of $\frac{1}{3} = \frac{3}{12}$ or $\frac{1}{4}$
$2 \times \frac{1}{3}$ of $\frac{3}{4} = \frac{6}{12}$	$\frac{3}{4}$ of $\frac{2}{3} = \frac{6}{12}$ or $\frac{1}{2}$
$3 \times \frac{1}{3}$ of $\frac{3}{4} = \frac{9}{12}$	$\frac{3}{4}$ of $\frac{3}{3} = \frac{9}{12}$ or $\frac{3}{4}$
$4 \times \frac{1}{3}$ of $\frac{3}{4} = \frac{12}{12}$	$\frac{3}{4}$ of $\frac{4}{3} = \frac{12}{12}$ or 1
$5 \times \frac{1}{3}$ of $\frac{3}{4} = \frac{15}{12}$	$\frac{3}{4}$ of $\frac{5}{3} = \frac{15}{12}$ or $\frac{5}{4} = 1\frac{1}{4}$.
&c.	&c.

And so on to the fractional parts of $\frac{5}{4}$, $\frac{6}{4}$, $\frac{7}{4}$, &c.

Second Square of the Fifth Line.

$\frac{1}{2}$ of $\frac{1}{5} = \frac{1}{10}$	$\frac{1}{5}$ of $\frac{1}{2} = \frac{1}{10}$
Twice $\frac{1}{2}$ of $\frac{1}{5} = 2 \times \frac{1}{10}$ or $\frac{2}{10}$	$\frac{1}{5}$ of $\frac{2}{2} = \frac{2}{10}$ or $\frac{1}{5}$
$3 \times \frac{1}{2}$ of $\frac{1}{5} = 3 \times \frac{1}{10}$ or $\frac{3}{10}$	$\frac{1}{5}$ of $\frac{3}{2} = \frac{3}{10}$
$4 \times \frac{1}{2}$ of $\frac{1}{5} = 4 \times \frac{1}{10}$ or $\frac{4}{10}$	$\frac{1}{5}$ of $\frac{4}{2} = \frac{4}{10}$ or $\frac{2}{5}$
$5 \times \frac{1}{2}$ of $\frac{1}{5} = 5 \times \frac{1}{10}$ or $\frac{5}{10}$	$\frac{1}{5}$ of $\frac{5}{2} = \frac{5}{10}$ or $\frac{1}{2}$

$\frac{1}{2}$ of $\frac{2}{5} = \frac{2}{10}$	$\frac{2}{5}$ of $\frac{1}{2} = \frac{2}{10}$ or $\frac{1}{5}$
Twice $\frac{1}{2}$ of $\frac{2}{5} = \frac{4}{10}$	$\frac{2}{5}$ of $\frac{2}{2} = \frac{4}{10}$ or $\frac{2}{5}$
$3 \times \frac{1}{2}$ of $\frac{2}{5} = \frac{6}{10}$	$\frac{2}{5}$ of $\frac{3}{2} = \frac{6}{10}$ or $\frac{3}{5}$
$4 \times \frac{1}{2}$ of $\frac{2}{5} = \frac{8}{10}$	$\frac{2}{5}$ of $\frac{4}{2} = \frac{8}{10}$ or $\frac{4}{5}$
&c.	&c.

$\frac{1}{2}$ of $\frac{3}{5} = \frac{3}{10}$	$\frac{3}{5}$ of $\frac{1}{2} = \frac{3}{10}$
$2 \times \frac{1}{2}$ of $\frac{3}{5} = \frac{6}{10}$	$\frac{3}{5}$ of $\frac{2}{2} = \frac{6}{10}$ or $\frac{3}{5}$
$3 \times \frac{1}{2}$ of $\frac{3}{5} = \frac{9}{10}$	$\frac{3}{5}$ of $\frac{3}{2} = \frac{9}{10}$
$4 \times \frac{1}{2}$ of $\frac{3}{5} = \frac{12}{10}$	$\frac{3}{5}$ of $\frac{4}{2} = \frac{12}{10}$ or $\frac{6}{5} = 1\frac{1}{5}$
$5 \times \frac{1}{2}$ of $\frac{3}{5} = \frac{15}{10}$	$\frac{3}{5}$ of $\frac{5}{2} = \frac{15}{10}$ or $1\frac{1}{2}$
&c.	&c.

And so on as before.

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* Read "Twice the third of one-fourth are twice one-twelfth or two-twelfths."

Fourth Square of the Sixth Line.

$\frac{1}{4}$ of $\frac{1}{6} = \frac{1}{24}$	$\frac{1}{6}$ of $\frac{1}{4} = \frac{1}{24}$
$2 \times \frac{1}{4}$ of $\frac{1}{6} = 2 \times \frac{1}{24}$ or $\frac{2}{24}$	$\frac{1}{6}$ of $\frac{2}{4} = \frac{2}{24}$ or $\frac{1}{12}$
$3 \times \frac{1}{4}$ of $\frac{1}{6} = 3 \times \frac{1}{24}$ or $\frac{3}{24}$	$\frac{1}{6}$ of $\frac{3}{4} = \frac{3}{24}$ or $\frac{1}{8}$
$4 \times \frac{1}{4}$ of $\frac{1}{6} = 4 \times \frac{1}{24}$ or $\frac{4}{24}$	$\frac{1}{6}$ of $\frac{4}{4} = \frac{4}{24}$ or $\frac{1}{6}$
$5 \times \frac{1}{4}$ of $\frac{1}{6} = 5 \times \frac{1}{24}$ or $\frac{5}{24}$	$\frac{1}{6}$ of $\frac{5}{4} = \frac{5}{24}$
$6 \times \frac{1}{4}$ of $\frac{1}{6} = 6 \times \frac{1}{24}$ or $\frac{6}{24}$	$\frac{1}{6}$ of $\frac{6}{4} = \frac{6}{24}$ or $\frac{1}{4}$
&c.	&c.

$\frac{1}{4}$ of $\frac{2}{6} = \frac{2}{24}$	$\frac{2}{6}$ of $\frac{1}{4} = \frac{2}{24}$ or $\frac{1}{12}$
$2 \times \frac{1}{4}$ of $\frac{2}{6} = \frac{4}{24}$	$\frac{2}{6}$ of $\frac{2}{4} = \frac{4}{24}$ or $\frac{1}{6}$
$3 \times \frac{1}{4}$ of $\frac{2}{6} = \frac{6}{24}$	$\frac{2}{6}$ of $\frac{3}{4} = \frac{6}{24}$ or $\frac{1}{4}$
$4 \times \frac{1}{4}$ of $\frac{2}{6} = \frac{8}{24}$	$\frac{2}{6}$ of $\frac{4}{4} = \frac{8}{24}$ or $\frac{1}{3}$
$5 \times \frac{1}{4}$ of $\frac{2}{6} = \frac{10}{24}$	$\frac{2}{6}$ of $\frac{5}{4} = \frac{10}{24}$ or $\frac{5}{12}$
&c.	&c.

$\frac{1}{4}$ of $\frac{3}{6} = \frac{3}{24}$	$\frac{3}{6}$ of $\frac{1}{4} = \frac{3}{24}$ or $\frac{1}{8}$
$2 \times \frac{1}{4}$ of $\frac{3}{6} = \frac{6}{24}$	$\frac{3}{6}$ of $\frac{2}{4} = \frac{6}{24}$ or $\frac{1}{4}$
$3 \times \frac{1}{4}$ of $\frac{3}{6} = \frac{9}{24}$	$\frac{3}{6}$ of $\frac{3}{4} = \frac{9}{24}$ or $\frac{3}{8}$
$4 \times \frac{1}{4}$ of $\frac{3}{6} = \frac{12}{24}$	$\frac{3}{6}$ of $\frac{4}{4} = \frac{12}{24}$ or $\frac{1}{2}$
&c.	&c.

In general we multiply by a fraction when we take the part or parts of the multiplicand indicated by the fractional multiplier. For example, in order to multiply $\frac{2}{3}$ by $\frac{4}{5}$, we have to take the fifth of $\frac{2}{3}$ four times; thus the fifth of $\frac{2}{3}$ is $\frac{2}{15}$, and four times the fifth of $\frac{2}{3}$ are four times $\frac{2}{15}$ or $\frac{8}{15}$. To multiply a fraction by a fraction, therefore, we have only to multiply the numerators by one another, to give the numerator of the product, and the multiplication of the denominators gives the denominator of the product. Hence the fraction of a fraction, is the same thing as the multiplication of a fraction by a fraction. This view of the multiplication of fractions is evidently a simple extension of the principle of the ordinary definition of multiplication; for adopting the common language, we might say, $\frac{2}{3}$ multiplied by $\frac{4}{5}$, is an operation whereby we take $\frac{2}{3}$ four-fifths of once.

This exercise also shows that we may divide a fraction

by a whole number in two ways; 1st, by dividing the numerator of the fraction by the divisor; and 2nd, by multiplying the denominator of the fraction by the divisor. Thus the fifth part of $\frac{2}{3}$ may be determined in two ways; 1st, $\frac{2}{3}$ are $\frac{10}{15}$, and the fifth of $\frac{10}{15}$ is $\frac{2}{15}$; 2nd, the fifth of $\frac{2}{3}$ is twice $\frac{1}{15}$ or $\frac{2}{15}$.

Questions on Case III., continued.

Third Square of the Fourth Line.

i. What is twice the $\frac{1}{3}$ of $\frac{5}{4}$?....*Ans.* $\frac{10}{12} = \frac{5}{6}$; because the third of $\frac{5}{4}$ is $\frac{5}{12}$ and twice the third of $\frac{5}{4}$ are twice $\frac{5}{12} = \frac{10}{12}$.

ii. What is $\frac{4}{3}$ of $\frac{5}{4}$?....*Ans.* $\frac{20}{12} = \frac{5}{3} = 1\frac{2}{3}$.

iii. What is $\frac{5}{4}$ of $\frac{2}{3}$?....*Ans.* $\frac{10}{12} = \frac{5}{6}$.

iv. Multiply $\frac{2}{3}$ by $1\frac{1}{4}$.....*Ans.* $\frac{10}{12} = \frac{5}{6}$.

v. Where shall I find $\frac{2}{3}$ of $\frac{3}{4}$?....*Ans.* On the third square of the fourth line; because I have there the thirds of fourths.

Fourth Square of the Fifth Line.

i. What is $\frac{2}{5}$ of $\frac{3}{4}$?....*Ans.* $\frac{6}{20} = \frac{3}{10}$; because the $\frac{1}{5}$ of $\frac{3}{4}$ is $\frac{3}{20}$, and therefore $\frac{2}{5}$ of $\frac{3}{4}$ twice $\frac{3}{20} = \frac{6}{20}$.

ii. Multiply $\frac{5}{4}$ by $\frac{3}{5}$.....*Ans.* $\frac{15}{20} = \frac{3}{4}$; because $\frac{1}{4}$ of $\frac{3}{5} = \frac{3}{20}$, and therefore $5 \times \frac{1}{4}$ of $\frac{3}{5} = 5$ times $\frac{3}{20} = \frac{15}{20}$.

&c., &c.

Second Square of the Third Line.

i. What is $\frac{3}{2}$ of $\frac{5}{3}$?....*Ans.* $\frac{15}{6} = 2\frac{1}{2}$; because the $\frac{1}{2}$ of $\frac{5}{3} = \frac{5}{6}$, and therefore $3 \times \frac{1}{2}$ of $\frac{5}{3} = \frac{15}{6}$.

ii. What is $\frac{2}{3}$ of $\frac{5}{2}$?....*Ans.* $\frac{10}{6} = \frac{5}{3} = 1\frac{2}{3}$.

iii. What is the product of $2\frac{1}{2}$ by $1\frac{1}{3}$?....*Ans.* $\frac{20}{6} = 3\frac{1}{3}$; because $2\frac{1}{2} = \frac{5}{2}$ and $1\frac{1}{3} = \frac{4}{3}$, therefore $\frac{4}{3}$ of $\frac{5}{2} = 4$ times $\frac{5}{6} = \frac{20}{6} = 3\frac{1}{3}$.

&c., &c.

Seventh Square of the Third Line.

i. What is $\frac{4}{7}$ of $\frac{2}{3}$?....*Ans.* $\frac{8}{21}$; because the seventh of $\frac{2}{3}$ is $\frac{2}{21}$, and 4 times $\frac{1}{7}$ of $\frac{2}{3}$ = 4 times $\frac{2}{21}$ or $\frac{8}{21}$.

ii. Multiply $1\frac{1}{3}$ by $2\frac{2}{7}$.....*Ans.* $\frac{64}{21} = 3\frac{1}{21}$.

iii. Multiply $\frac{5}{3}$ by $1\frac{1}{7}$.....*Ans.* $\frac{40}{21} = 1\frac{19}{21}$.

iv. Multiply $\frac{1}{3}$ of 2 by $\frac{2}{7}$.....*Ans.* $\frac{4}{21}$.

v. Multiply $1\frac{1}{7}$ by $\frac{2}{3}$ of 4.....*Ans.* $\frac{64}{21} = 3\frac{1}{21}$.

Similar questions may be proposed on any other square.

Money, Weights, and Measures.

Fifth Square, Fourth Line.

i. How much is $\frac{3}{5}$ of $\frac{1}{4}$ of 3*s*. 4*d*.?....*Ans.* 6*d*.; because $\frac{3}{5}$ of $\frac{1}{4}$ are $\frac{3}{20}$; 3*s*. 4*d*. contain 40*d*., and $\frac{3}{20}$ of 40*d*. are 6*d*.

ii. If a pound of cheese cost $7\frac{1}{2}d.$, what is the value of $\frac{2}{5}$ of $\frac{1}{4}$ of a lb?....*Ans.* $\frac{3}{4}d.$; because $\frac{2}{5}$ of $\frac{1}{4}$ are $\frac{2}{20}$, and $\frac{2}{20}$ of $7\frac{1}{2}d.$ are $\frac{3}{4}d.$

Ninth Square, Second Line.

i. What is the $\frac{1}{9}$ of $\frac{1}{2}$ of 3*s*.?....*Ans.* 2*d*.

ii. If the cost of an article amount to 1*l*. 16*s*., what is the value of the remainder when the half of one-ninth is taken away?....*Ans.* 1*l*. 14*s*.; because the $\frac{1}{2}$ of $\frac{1}{9}$ is $\frac{1}{18}$; 1*l*. 16*s*. contains 36*s*.; the $\frac{1}{18}$ of 36*s*. is 2*s*.; then 1*l*. 16*s*. – 2*s*. = 1*l*. 14*s*.

iii. How many shillings are contained in $\frac{2}{9}$ of $\frac{3}{2}$ of a guinea?....*Ans.* 7*s*.; because $\frac{2}{9}$ of $\frac{3}{2}$ are $\frac{6}{18}$ or $\frac{1}{3}$, and the $\frac{1}{3}$ of a guinea is 7*s*.

The pupil having been taught how to find the value of compound fractions, he will now be enabled to add and subtract such fractions. The solutions of the following questions become remarkably simple, if the squares referred to be placed before the pupils on an enlarged scale.

Questions on the Addition and Subtraction of Compound Fractions.

Third Square, Fourth Line.

i. In twice $\frac{1}{3}$ and 3 times the $\frac{1}{4}$ of $\frac{1}{3}$, how many twelfths? ....*Ans.* $\frac{11}{12}$; because $\frac{1}{3}$ contains $\frac{4}{12}$, therefore $\frac{2}{3}$ will contain twice $\frac{4}{12} = \frac{8}{12}$ and $3 \times \frac{1}{4}$ of $\frac{1}{3} = \frac{3}{12}$, hence $\frac{8}{12} + \frac{3}{12} = \frac{11}{12}$.

ii. In the $\frac{1}{4}$ of $\frac{1}{3}$ and twice the $\frac{1}{3}$ of $\frac{3}{4}$, how many twelfths? ....*Ans.* $\frac{7}{12}$.

iii. How many twelfths are there in the $\frac{1}{3}$ of $\frac{2}{4}$ and twice $\frac{1}{6}$ of $\frac{1}{2}$?....*Ans.* $\frac{4}{12}$.

iv. What is the difference between twice the $\frac{1}{3}$ of $\frac{1}{4}$ and once the $\frac{1}{4}$ of $\frac{1}{3}$?....*Ans.* $\frac{1}{12}$.

Second Square, Third Line.

i. How many sixths are there in the $\frac{1}{3}$ of $\frac{1}{2}$ and the $\frac{1}{2}$ of $\frac{1}{3}$?....*Ans.* $\frac{2}{6}$.

ii. How many sixths are there in 4 times the $\frac{1}{2}$ of $\frac{1}{3}$ and twice the $\frac{1}{3}$ of $\frac{1}{2}$?....*Ans.* $\frac{6}{6}$.

iii. What is the difference in sixths between twice the third of $\frac{1}{2}$, and the $\frac{1}{2}$ of $\frac{1}{3}$?....*Ans.* $\frac{1}{6}$.

iv. What is the difference between the $\frac{1}{2}$ of $\frac{1}{3}$ and the $\frac{1}{3}$ of $\frac{1}{2}$?....*Ans.* Nothing.

v. How many sixths are 3 times $\frac{1}{2}$, the $\frac{1}{3}$ of $\frac{1}{2}$, and the $\frac{1}{2}$ of $\frac{1}{3}$?....*Ans.* $\frac{11}{6}$.

Fifth Square, Third Line.

i. How many fifteenths are there in twice $\frac{1}{5}$ and 3 times the third of $\frac{1}{5}$?....*Ans.* $\frac{9}{15}$.

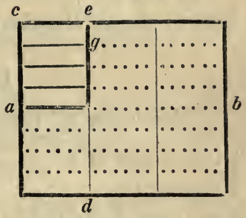
ii. How many fifteenths are there in $\frac{1}{3}$ of $\frac{1}{5}$ and twice the $\frac{1}{5}$ of $\frac{1}{3}$?....*Ans.* $\frac{3}{15}$.

iii. How many fifteenths are there in $\frac{1}{5}$ of $\frac{1}{3}$ and 4 times the $\frac{1}{3}$ of $\frac{1}{5}$?....*Ans.* $\frac{5}{15}$.

iv. How many fifteenths are there in $\frac{2}{3}$, $\frac{3}{5}$, and twice the $\frac{1}{5}$ of $\frac{1}{3}$?....*Ans.* $\frac{21}{15}$.

And so on to any other square.

Miscellaneous Questions.



i. Give a representation upon your slates of the $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{1}{3}$....*Ans.* The line *e d* cuts off the $\frac{1}{3}$ of the square or unit; *a b* divides each third into two equal parts, so that the space *a e* represents the $\frac{1}{2}$ of $\frac{1}{3}$, and finally *c g* is one of the four parts into which *a e* is divided, that is, *c g* is the $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{1}{3}$, which is obviously equal to the $\frac{1}{24}$ part of the whole square or unit.

ii. Divide this rod so as to represent the $\frac{1}{3}$ of $\frac{1}{2}$ of $\frac{3}{4}$.

iii. Cut this apple so as to represent $\frac{2}{3}$ of $\frac{1}{2}$ of $\frac{5}{6}$.

SEVENTH EXERCISE.

Multiplication by Mixed Numbers, and Division of Fractions.

The first column of this exercise shows how fractions may be resolved into parts of other fractions; and the second column exhibits the reproduction of the original fraction. The former operation is equivalent to the division of one fraction by another; the latter enables the pupil to perform the multiplication of a fraction by a mixed number.

The processes of this exercise are performed with the aid of both Boards of Fractions.

I. *On the Board of Simple Fractions.*

It will be observed that the present exercise is similar in form to that of the Fourth Exercise in Book I.

Line of Thirds.

Resolution of Thirds into parts of $\frac{2}{3}$, and conversely.

$\frac{1}{3}$ is the half of $\frac{2}{3}$	The half of $\frac{2}{3}$ is $\frac{1}{3}$
$\frac{2}{3}$ are once $\frac{2}{3}$	Once $\frac{2}{3}$ are $\frac{2}{3}$
$\frac{3}{3}$ are once $\frac{2}{3}$ and the half of $\frac{2}{3}$	Once $\frac{2}{3}$ and $\frac{1}{2}$ of $\frac{2}{3}$ are $\frac{3}{3}$
$\frac{4}{3}$ are twice $\frac{2}{3}$	Twice $\frac{2}{3}$ are $\frac{4}{3}$
$\frac{5}{3}$ are twice $\frac{2}{3}$ and the $\frac{1}{2}$ of $\frac{2}{3}$	Twice $\frac{2}{3}$ and $\frac{1}{2}$ of $\frac{2}{3}$ are $\frac{5}{3}$
&c.	&c.

Resolution of Thirds into parts of $\frac{3}{3}$, and conversely.

$\frac{1}{3}$ is the third of $\frac{3}{3}$	$\frac{1}{3}$ of $\frac{3}{3}$ is $\frac{1}{3}$
$\frac{2}{3}$ are twice the third of $\frac{3}{3}$	Twice $\frac{1}{3}$ of $\frac{3}{3}$ are $\frac{2}{3}$
$\frac{3}{3}$ are once $\frac{3}{3}$	Once $\frac{3}{3}$ are $\frac{3}{3}$
$\frac{4}{3}$ are once $\frac{3}{3}$ and $\frac{1}{3}$ of $\frac{3}{3}$	Once $\frac{3}{3}$ and $\frac{1}{3}$ of $\frac{3}{3}$ are $\frac{4}{3}$
$\frac{5}{3}$ are once $\frac{3}{3}$ and twice $\frac{1}{3}$ of $\frac{3}{3}$	Once $\frac{3}{3}$ and twice $\frac{1}{3}$ of $\frac{3}{3}$ are $\frac{5}{3}$
&c.	&c.

Resolution of Thirds into parts of $\frac{4}{3}$, and conversely.

$\frac{1}{3}$ is the fourth of $\frac{4}{3}$	The $\frac{1}{4}$ of $\frac{4}{3}$ is $\frac{1}{3}$
$\frac{2}{3}$ are twice $\frac{1}{4}$ of $\frac{4}{3}$	Twice the $\frac{1}{4}$ of $\frac{4}{3}$ are $\frac{2}{3}$
$\frac{3}{3}$ are 3 times $\frac{1}{4}$ of $\frac{4}{3}$	3 times $\frac{1}{4}$ of $\frac{4}{4}$ are $\frac{3}{3}$
$\frac{4}{3}$ are once $\frac{4}{3}$	Once $\frac{4}{3}$ are $\frac{4}{3}$
$\frac{5}{3}$ are once $\frac{4}{3}$ and the $\frac{1}{4}$ of $\frac{4}{3}$	Once $\frac{4}{3}$ and $\frac{1}{4}$ of $\frac{4}{3}$ are $\frac{5}{3}$
$\frac{6}{3}$ are once $\frac{4}{3}$ and twice $\frac{1}{4}$ of $\frac{4}{3}$	Once $\frac{4}{3}$ and twice $\frac{1}{3}$ of $\frac{4}{3}$ are $\frac{6}{3}$
&c.	&c.

And so on to the resolution of thirds into $\frac{5}{3}$, $\frac{6}{3}$, $\frac{7}{3}$, $\frac{8}{3}$, &c. and conversely.

Questions on the Seventh Exercise.

On the Board of Simple Fractions.

Line of Halves.

i. Once $\frac{3}{2}$ and the $\frac{1}{3}$ of $\frac{3}{2}$.....*Ans.* $\frac{4}{2}$ or 2.

ii. 3 times $\frac{5}{2}$ and the $\frac{1}{5}$ of $\frac{5}{2}$.....*Ans.* 8.

iii. $\frac{7}{2}$ contain how many times $\frac{3}{2}$?....*Ans.* $2\frac{1}{3}$ times; because $\frac{7}{2}$ are twice $\frac{3}{2}$ and the $\frac{1}{3}$ of $\frac{3}{2}$.

Line of Thirds.

i. How many two-thirds can be taken out of $\frac{7}{3}$?....*Ans.* $3\frac{1}{2}$ times; because $\frac{7}{3}$ are 3 times $\frac{2}{3}$ and the $\frac{1}{2}$ of $\frac{2}{3}$.

ii. 3 times $\frac{4}{3}$ and the half of $\frac{4}{3}$.....*Ans.* $\frac{14}{3}$ or $4\frac{2}{3}$.

iii. 3 times $\frac{5}{3}$ and twice the $\frac{1}{5}$ of $\frac{5}{3}$.....*Ans.* $\frac{17}{3}$ or $5\frac{2}{3}$.

Line of Fourths.

i. 3 times $\frac{5}{4}$ and 3 times the $\frac{1}{5}$ of $\frac{5}{4}$.....*Ans.* $\frac{18}{4}$ or $4\frac{1}{2}$.

ii. How many three-fourths can be taken out of $\frac{11}{4}$?.... *Ans.* $\frac{3}{4}$ are contained $3\frac{2}{3}$ times in $\frac{11}{4}$; because $\frac{11}{4}$ are 3 times $\frac{3}{4}$ and twice the $\frac{1}{3}$ of $\frac{3}{4}$.

Line of Fifths.

i. Twice $\frac{2}{5}$ and the $\frac{1}{2}$ of $\frac{2}{5}$.....*Ans.* $\frac{5}{5}$ or 1.

ii. How many times $\frac{3}{5}$ are $\frac{7}{5}$?....*Ans.* $2\frac{1}{3}$ times; because $\frac{7}{5}$ are twice $\frac{3}{5}$ and the $\frac{1}{3}$ of $\frac{3}{5}$.

Similar questions may be given on the other lines.

Miscellaneous Questions.

i. Multiply $\frac{6}{5}$ by $1\frac{1}{2}$.....*Ans.* $\frac{9}{5}$.

Proof. Once $\frac{6}{5}$ and $\frac{1}{2}$ of $\frac{6}{5}$ are $\frac{6}{5}$ and $\frac{3}{5} = \frac{9}{5}$.

ii. Multiply $\frac{8}{3}$ by $2\frac{1}{4}$.....*Ans.* $\frac{18}{3}$ or 6.

Proof. Twice $\frac{8}{3}$ and $\frac{1}{4}$ of $\frac{8}{3}$ are $\frac{16}{3}$ and $\frac{2}{3} = \frac{18}{3}$ or 6.

iii. Multiply $\frac{2}{5}$ by $1\frac{1}{2}$.....*Ans.* $\frac{3}{5}$.

iv. Multiply $\frac{9}{7}$ by $1\frac{2}{3}$.....*Ans.* $\frac{15}{7} = 2\frac{1}{7}$.

Proof. $\frac{2}{3}$ of $\frac{9}{7}$ are twice $\frac{3}{7} = \frac{6}{7}$; and therefore once $\frac{9}{7}$ and twice the third of $\frac{9}{7}$ are $\frac{9}{7} + \frac{6}{7} = \frac{15}{7}$.

v. What is the product of $\frac{14}{5}$ by $2\frac{1}{7}$?....*Ans.* $\frac{30}{5} = 6$.

Proof. Twice $\frac{14}{5}$ and the $\frac{1}{7}$ of $\frac{14}{5}$ are $\frac{28}{5} + \frac{2}{5} = \frac{30}{5}$ or 6.

vi. What is the product of $\frac{3}{4}$ by $3\frac{2}{3}$?....*Ans.* $\frac{11}{4}$.

Proof. Three times $\frac{3}{4}$ and twice the $\frac{1}{3}$ of $\frac{3}{4}$ are $\frac{9}{4} + \frac{2}{4} = \frac{11}{4}$.

vii. Multiply the $\frac{1}{2}$ of $\frac{3}{4}$ by $1\frac{1}{3}$.....*Ans.* $\frac{4}{8} = \frac{1}{2}$.

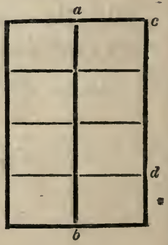
Proof. $\frac{1}{2}$ of $\frac{3}{4} = \frac{3}{8}$; and once $\frac{3}{8}$ and the third of $\frac{3}{8}$ are $\frac{4}{8} = \frac{1}{2}$.

viii. Resolve $\frac{11}{8}$ into parts of $\frac{3}{8}$.....*Ans.* $\frac{11}{8}$ are 3 times $\frac{3}{8}$ and twice $\frac{1}{3}$ of $\frac{3}{8}$.

II. *On the Board of Compound Fractions.*

The mode of proceeding with this exercise on the Compound Board, may be illustrated by the following example :

Fourth Square, Second Line.



Suppose the annexed figure to represent a unit divided into halves by a vertical line, and into fourths by horizontal lines. By placing one pointer along the line *a b*, and the other along the line *c d*, the teacher will mark out the space *a b c d*, by which it is shown that "$\frac{7}{8}$ are once $\frac{1}{2}$ and 3 times $\frac{1}{4}$ of $\frac{1}{2}$," and conversely, "once $\frac{1}{2}$ and 3 times $\frac{1}{4}$ of $\frac{1}{2}$ are $\frac{7}{8}$."

Resolution of Eighths into parts of $\frac{1}{4}$, and conversely.

$\frac{1}{8}$ is the $\frac{1}{2}$ of $\frac{1}{4}$	The $\frac{1}{2}$ of $\frac{1}{4}$ is $\frac{1}{8}$
$\frac{2}{8}$ are once $\frac{1}{4}$	Once $\frac{1}{4}$ are $\frac{2}{8}$
$\frac{3}{8}$ are once $\frac{1}{4}$ and $\frac{1}{2}$ of $\frac{1}{4}$	Once $\frac{1}{4}$ and $\frac{1}{2}$ of $\frac{1}{4}$ are $\frac{3}{8}$
$\frac{4}{8}$ are twice $\frac{1}{4}$	Twice $\frac{1}{4}$ are $\frac{4}{8}$
$\frac{5}{8}$ are twice $\frac{1}{4}$ and $\frac{1}{2}$ of $\frac{1}{4}$	Twice $\frac{1}{4}$ and $\frac{1}{2}$ of $\frac{1}{4}$ are $\frac{5}{8}$
&c.	&c.

Resolution of Eighths into parts of $\frac{2}{4}$, and conversely.

$\frac{1}{8}$ is the $\frac{1}{4}$ of $\frac{2}{4}$	The $\frac{1}{4}$ of $\frac{2}{4}$ is $\frac{1}{8}$
$\frac{2}{8}$ are twice $\frac{1}{4}$ of $\frac{2}{4}$	Twicc $\frac{1}{4}$ of $\frac{2}{4}$ are $\frac{2}{8}$
$\frac{3}{8}$ are 3 times $\frac{1}{4}$ of $\frac{2}{4}$	3 times $\frac{1}{4}$ of $\frac{2}{4}$ are $\frac{3}{8}$
$\frac{4}{8}$ are once $\frac{2}{4}$	Once $\frac{2}{4}$ are $\frac{4}{8}$
$\frac{5}{8}$ are once $\frac{2}{4}$ and $\frac{1}{4}$ of $\frac{2}{4}$	Once $\frac{2}{4}$ and $\frac{1}{4}$ of $\frac{2}{4}$ are $\frac{5}{8}$
$\frac{6}{8}$ are once $\frac{2}{4}$ and twice $\frac{1}{4}$ of $\frac{2}{4}$	Once $\frac{2}{4}$ and twice $\frac{1}{4}$ of $\frac{2}{4}$ are $\frac{6}{8}$
$\frac{7}{8}$ are once $\frac{2}{4}$ and $3\times\frac{1}{4}$ of $\frac{2}{4}$	Once $\frac{2}{4}$ and $3\times\frac{1}{4}$ of $\frac{2}{4}$ are $\frac{7}{8}$
&c.	&c.

Resolution of Eighths into parts of $\frac{3}{4}$, and conversely.

$\frac{1}{8}$ is $\frac{1}{6}$ of $\frac{3}{4}$	$\frac{1}{6}$ of $\frac{3}{4}$ is $\frac{1}{8}$
$\frac{2}{8}$ are twice $\frac{1}{6}$ of $\frac{3}{4}$	Twice $\frac{1}{6}$ of $\frac{3}{4}$ are $\frac{2}{8}$
$\frac{3}{8}$ are 3 times $\frac{1}{6}$ of $\frac{3}{4}$	3 times $\frac{1}{6}$ of $\frac{3}{4}$ are $\frac{3}{8}$
$\frac{4}{8}$ are $4\times\frac{1}{6}$ of $\frac{3}{4}$	$4\times\frac{1}{6}$ of $\frac{3}{4}$ are $\frac{4}{8}$
$\frac{5}{8}$ are $5\times\frac{1}{6}$ of $\frac{3}{4}$	$5\times\frac{1}{6}$ of $\frac{3}{4}$ are $\frac{5}{8}$
$\frac{6}{8}$ are once $\frac{3}{4}$	Once $\frac{3}{4}$ are $\frac{6}{8}$
$\frac{7}{8}$ are once $\frac{3}{4}$ and $\frac{1}{6}$ of $\frac{3}{4}$	Once $\frac{3}{4}$ and $\frac{1}{6}$ of $\frac{3}{4}$ are $\frac{7}{8}$
$\frac{8}{8}$ are once $\frac{3}{4}$ and twice $\frac{1}{6}$ of $\frac{3}{4}$	Once $\frac{3}{4}$ and twice $\frac{1}{6}$ of $\frac{3}{4}$ are $\frac{8}{8}$
&c.	&c.

And so on to steps beginning with

$\frac{1}{8}$ is $\frac{1}{8}$ of $\frac{4}{4}$	$\frac{1}{8}$ of $\frac{4}{4}$ is $\frac{1}{8}$

$\frac{1}{8}$ is $\frac{1}{10}$ of $\frac{5}{4}$	$\frac{1}{10}$ of $\frac{5}{4}$ is $\frac{1}{8}$

$\frac{1}{8}$ is $\frac{1}{12}$ of $\frac{6}{4}$	$\frac{1}{12}$ of $\frac{6}{4}$ is $\frac{1}{8}$
&c.	&c.

Resolution of Eighths into parts of $\frac{1}{2}$, and conversely.

$\frac{1}{8}$ is $\frac{1}{4}$ of $\frac{1}{2}$	$\frac{1}{4}$ of $\frac{1}{2}$ is $\frac{1}{8}$
$\frac{2}{8}$ are twice $\frac{1}{4}$ of $\frac{1}{2}$	Twice $\frac{1}{4}$ of $\frac{1}{2}$ are $\frac{2}{8}$
$\frac{3}{8}$ are 3 times $\frac{1}{4}$ of $\frac{1}{2}$	3 times $\frac{1}{4}$ of $\frac{1}{2}$ are $\frac{3}{8}$
$\frac{4}{8}$ are once $\frac{1}{2}$	Once $\frac{1}{2}$ are $\frac{4}{8}$
$\frac{5}{8}$ are once $\frac{1}{2}$ and $\frac{1}{4}$ of $\frac{1}{2}$	Once $\frac{1}{2}$ and $\frac{1}{4}$ of $\frac{1}{2}$ are $\frac{5}{8}$
$\frac{6}{8}$ are once $\frac{1}{2}$ and twice $\frac{1}{4}$ of $\frac{1}{2}$	Once $\frac{1}{2}$ and twice $\frac{1}{4}$ of $\frac{1}{2}$ are $\frac{6}{8}$
&c. &c.	&c. &c.

And so on as before to the steps beginning with

$\frac{1}{8}$ is $\frac{1}{8}$ of $\frac{2}{2}$	$\frac{1}{8}$ of $\frac{2}{2}$ is $\frac{1}{8}$
$\frac{1}{8}$ is $\frac{1}{12}$ of $\frac{3}{2}$	$\frac{1}{12}$ of $\frac{3}{2}$ is $\frac{1}{8}$
$\frac{1}{8}$ is $\frac{1}{16}$ of $\frac{4}{2}$	$\frac{1}{16}$ of $\frac{4}{2}$ is $\frac{1}{8}$
&c.	&c.

In the same way we may exhibit the resolution and composition of any number of fourths and halves, and the same principle will apply to any square taken on any other line.

By this exercise we are taught to perform multiplication by another process, which shows that the result is the same whether we multiply $\frac{2}{3}$ by $3\frac{1}{2}$ according to the common acceptation of multiplication, or multiply these quantities together in the form of fractions, as explained in the preceding exercise; thus $3\frac{1}{2}$ times $\frac{2}{3}=3\times\frac{2}{3}$ and $\frac{1}{2}$ of $\frac{2}{3}=\frac{7}{3}$: in the other case $3\frac{1}{2}=\frac{7}{2}$, and $\frac{7}{2}$ of $\frac{2}{3}=\frac{14}{6}$ or $\frac{7}{3}$, which result is the same as that obtained by the first process.

The division of fractions is here deduced by a process somewhat indirect, but the method of ratios employed in the succeeding exercise, will, it is believed, be found more general and practical. The multiplication of fractions can, in most cases, be better performed by reducing the mixed number to the form of an improper fraction, and then proceeding as in the sixth exercise.

Questions on the Seventh Exercise.

On the Board of Compound Fractions.

Fourth Square, Third Line.

i. What is the product of $\frac{1}{4}$ by $1\frac{1}{3}$?....*Ans.* $\frac{4}{12} = \frac{1}{3}$.

ii. What is the product of $\frac{1}{4}$ by $1\frac{2}{3}$?....*Ans.* $\frac{5}{12}$.

Proof. Twice the third of $\frac{1}{4}$ are twice $\frac{1}{12} = \frac{2}{12}$; once $\frac{1}{4}$ is $\frac{3}{12}$; and once $\frac{1}{4}$ and twice the third of $\frac{1}{4}$ are $\frac{3}{12} + \frac{2}{12} = \frac{5}{12}$.

iii. Multiply $\frac{2}{3}$ by $1\frac{1}{4}$.....*Ans.* $\frac{10}{12} = \frac{5}{6}$.

Proof. Once $\frac{2}{3}$ and the $\frac{1}{4}$ of $\frac{2}{3}$ are $\frac{10}{12}$ or $\frac{5}{6}$.

iv. How many twelfths are there in once $\frac{1}{3}$ and 3 times the fourth of $\frac{1}{3}$?....*Ans.* $\frac{7}{12}$.

Proof. Once $\frac{1}{3}$ are $\frac{4}{12}$, $3 \times \frac{1}{4}$ of $\frac{1}{3} = \frac{3}{12}$; and therefore $\frac{4}{12}$ and $\frac{3}{12} = \frac{7}{12}$.

v. Resolve $\frac{9}{12}$ into parts of $\frac{2}{3}$?....*Ans.* $\frac{9}{12}$ are once $\frac{2}{3}$ and $\frac{1}{8}$ of $\frac{2}{3}$.

Third Square, Second Line.

i. Multiply $\frac{1}{3}$ by $2\frac{1}{2}$.....*Ans.* $\frac{5}{6}$.

Proof. Twice $\frac{1}{3}$ and the $\frac{1}{2}$ of $\frac{1}{3}$ are $\frac{5}{6}$.

ii. How many sixths are there in $3\frac{1}{3}$ times $\frac{1}{2}$?....*Ans.* $\frac{10}{6}$; 3 times $\frac{1}{2}$ and the third of $\frac{1}{2}$ are $\frac{9}{6}$ and $\frac{1}{6} = \frac{10}{6}$.

iii. How many sixths are there in $2\frac{1}{3}$ times $\frac{3}{2}$?....*Ans.* $\frac{21}{6}$.

iv. How many sixths are there in 3 times $\frac{2}{2}$ and twice the third of $\frac{2}{2}$?....*Ans.* $\frac{22}{6}$.

v. Resolve $\frac{5}{6}$ into $\frac{1}{2}$.....*Ans.* $\frac{5}{6}$ are once $\frac{1}{2}$ and twice the third of $\frac{1}{2}$.

vi. Resolve $\frac{8}{6}$ into parts of $\frac{2}{2}$.....*Ans.* $\frac{8}{6}$ are once $\frac{2}{2}$ and twice the $\frac{1}{6}$ of $\frac{2}{2}$.

vii. How many times $\frac{2}{3}$ are there contained in $\frac{11}{6}$?....*Ans.* $\frac{11}{6}$ are twice $\frac{2}{3}$ and 3 times $\frac{1}{4}$ of $\frac{2}{3}$, that is, $2\frac{3}{4}$ times $\frac{2}{3}$ are $\frac{11}{6}$.

viii. How many thirds can you take out of $\frac{5}{6}$?....*Ans.* $2\frac{1}{2}$, because $\frac{5}{6}$ are twice $\frac{1}{3}$ and the $\frac{1}{2}$ of $\frac{1}{3}$.

ix. How many sixths are there in $\frac{4}{3}$?....*Ans.* 8; because $\frac{1}{4}$ contains $\frac{2}{6}$, and therefore $\frac{4}{3}$ will contain 4 times $\frac{2}{6}$ or $\frac{8}{6}$.

x. How many two-thirds can you take out of $\frac{9}{6}$?....*Ans.* $2\frac{1}{4}$; because $\frac{9}{6}$ are twice $\frac{2}{3}$ and the $\frac{1}{4}$ of $\frac{2}{3}$.

Second Square, Fifth Line.

i. How many times are $\frac{3}{10}$ contained in $\frac{1}{2}$?....*Ans.* Once and $\frac{2}{3}$ times; because $\frac{1}{2}$ are once $\frac{3}{10}$ and twice the $\frac{1}{3}$ of $\frac{3}{10}$.

ii. Three times $\frac{3}{2}$ and $\frac{1}{5}$ of $\frac{3}{2}$.....*Ans.* $\frac{48}{10}$ or $4\frac{4}{5}$.

iii. How many seven-tenths can you take out of 1?....*Ans.* Once and $\frac{3}{7}$ times; because $\frac{2}{2}$ are once $\frac{7}{10}$ and 3 times the $\frac{1}{7}$ of $\frac{7}{10}$.

EIGHTH EXERCISE.

Ratios of Fractional Numbers.

The preceding exercises having been properly taught, the pupil will have learnt, that we add, subtract, and repeat fractional units, in the same manner as we add, subtract, and repeat simple units. It will now be shown that we find the ratio of any two groups of fractional units in the same manner as in the Fifth Exercise of Book I. was found the ratio of any two groups of simple units.

It will be convenient to divide the subject of fractional ratios into two cases: 1st, When two fractions are given to find their ratio; and 2nd, When a given fraction is a given ratio of a fraction required. The two Boards of Fractions will be used in succession in illustrating each case.

Case I. *When two fractions are given, to find their ratio.*

On the Board of Simple Fractions.

As an illustration of the mode of proceeding with the following tables, suppose the teacher is about to exhibit the ratio of $\frac{5}{3}$ to any other number of thirds. When the teacher places one of his pointers upon 5-thirds; the pupils say, "5-thirds are 5 times $\frac{1}{3}$;" without removing this pointer, the teacher places another pointer upon $\frac{2}{3}$; the pupils then say, "$\frac{2}{3}$ are twice $\frac{1}{3}$, 5 times $\frac{1}{3}$ are 5 times the half of twice $\frac{1}{3}$." Still keeping the first pointer in its place, the teacher removes the second to 3-thirds; the pupils then say, "$\frac{3}{3}$ are 3 times $\frac{1}{3}$, 5 times $\frac{1}{3}$ are 5 times the third of 3 times $\frac{1}{3}$," and so on.

In order that the principle may be thoroughly understood, the teacher may, occasionally, propose such questions as the following:

Teacher. What have you just proved in reference to $\frac{5}{3}$?

Pupil. That $\frac{5}{3}$ are 5 times the third of $\frac{3}{3}$.

Teacher. Why?

Pupil. Because the third of $\frac{3}{3}$, which is $\frac{1}{3}$, repeated 5 times produces $\frac{5}{3}$.

Line of Halves.

Ratio of $\frac{1}{2}$ to any given number of halves.

$\frac{1}{2}$ is the half of twice $\frac{1}{2}$ or $\frac{2}{2}$.

$\frac{1}{2}$ is the third of 3 times $\frac{1}{2}$ or $\frac{3}{2}$.

$\frac{1}{2}$ is the fourth of 4 times $\frac{1}{2}$ or $\frac{4}{2}$.

And so on.

Ratio of $\frac{2}{2}$ to any given number of halves.

$\frac{2}{2}$ are twice $\frac{1}{2}$*.

$\frac{3}{2}$ are 3 times $\frac{1}{2}$. Twice $\frac{1}{2}$ are twice the third of 3 times $\frac{1}{2}$.

$\frac{4}{2}$ are 4 times $\frac{1}{2}$. Twice $\frac{1}{2}$ are twice the fourth of 4 times $\frac{1}{2}$.

$\frac{5}{2}$ are 5 times $\frac{1}{2}$. Twice $\frac{1}{2}$ are twice the fifth of 5 times $\frac{1}{2}$.

And so on.

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* In some cases the teacher may find it necessary, at every successive ratio, to repeat the decomposition given at the head of each table.

Ratio of $\frac{3}{2}$ to any given number of halves.

$\frac{3}{2}$ are 3 times $\frac{1}{2}$.

$\frac{2}{2}$ are twice $\frac{1}{2}$. 3 times $\frac{1}{2}$ are 3 times the half of twice $\frac{1}{2}$.

$\frac{4}{2}$ are 4 times $\frac{1}{2}$. 3 times $\frac{1}{2}$ are 3 times the fourth of 4 times $\frac{1}{2}$.

$\frac{5}{2}$ are 5 times $\frac{1}{2}$. 3 times $\frac{1}{2}$ are 3 times the fifth of 5 times $\frac{1}{2}$.

And so on.

Ratio of $\frac{4}{2}$ to any given number of halves.

$\frac{4}{2}$ are 4 times $\frac{1}{2}$.

$\frac{2}{2}$ are twice $\frac{1}{2}$. 4 times $\frac{1}{2}$ are 4 times the half of twice $\frac{1}{2}$.

$\frac{3}{2}$ are 3 times $\frac{1}{2}$. 4 times $\frac{1}{2}$ are 4 times the third of 3 times $\frac{1}{2}$.

And so on.

The teacher may then go on to exhibit the ratio of $\frac{5}{2}$, $\frac{6}{2}$, $\frac{7}{2}$, $\frac{8}{2}$, $\frac{9}{2}$, &c., to any given number of halves.

Although the foregoing tables will always give us the ratio of any two fractions, yet it is evident that this ratio will not be in the lowest terms when the numerators of the fractions compared contain a common factor; the following form is constructed to obviate this objection. A few examples will suffice to show the features of the method.

Reduction of Ratios.

Line of Halves.

Ratio of $\frac{3}{2}$ to the multiples of $\frac{3}{2}$.

$\frac{6}{2}$ are twice $\frac{3}{2}$.
$\frac{3}{2}$ are the half of twice $\frac{3}{2}$.
$\frac{9}{2}$ are 3 times $\frac{3}{2}$.
$\frac{3}{2}$ are the third of 3 times $\frac{3}{2}$.
$\frac{12}{2}$ are 4 times $\frac{3}{2}$.
$\frac{3}{2}$ are the fourth of 4 times $\frac{3}{2}$.

And so on.

Ratio of $\frac{5}{2}$ to the multiples of $\frac{5}{2}$.

$\frac{10}{2}$ are twice $\frac{5}{2}$.
$\frac{5}{2}$ are the half of twice $\frac{5}{2}$.
$\frac{15}{2}$ are 3 times $\frac{5}{2}$.
$\frac{5}{2}$ are the third of 3 times $\frac{5}{2}$.

And so on.

Ratio of $\frac{6}{2}$ to the multiples of $\frac{3}{2}$.

$\frac{6}{2}$ are twice $\frac{3}{2}$.

$\frac{9}{2}$ are 3 times $\frac{3}{2}$. Twice $\frac{3}{2}$ are twice the third of 3 times $\frac{3}{2}$.

$\frac{12}{2}$ are 4 times $\frac{3}{2}$. Twice $\frac{3}{2}$ are twice the fourth of 4 times $\frac{3}{2}$.

And so on.

Ratio of $\frac{9}{2}$ to the multiples of $\frac{3}{2}$.

$\frac{9}{2}$ are 3 times $\frac{3}{2}$.

$\frac{6}{2}$ are twice $\frac{3}{2}$. 3 times $\frac{3}{2}$ are 3 times the half of twice $\frac{3}{2}$.

$\frac{12}{2}$ are 4 times $\frac{3}{2}$. 3 times $\frac{3}{2}$ are 3 times the fourth of 4 times $\frac{3}{2}$.

And so on.

And so on to other combinations.

Third Line.

Ratio of one-third to any number of thirds.

$\frac{1}{3}$ is the half of twice $\frac{1}{3}$ or $\frac{2}{3}$.

$\frac{1}{3}$ is the third of 3 times $\frac{1}{3}$ or $\frac{3}{3}$.

$\frac{1}{3}$ is the fourth of 4 times $\frac{1}{3}$ or $\frac{4}{3}$.

$\frac{1}{3}$ is the fifth of 5 times $\frac{1}{3}$ or $\frac{5}{3}$.

And so on.

Ratio of $\frac{2}{3}$ to any number of thirds.

$\frac{2}{3}$ are twice $\frac{1}{3}$. Twice $\frac{1}{3}$ are twice the half of twice $\frac{1}{3}$.

$\frac{3}{3}$ are 3 times $\frac{1}{3}$. Twice $\frac{1}{3}$ are twice the third of 3 times $\frac{1}{3}$.

$\frac{4}{3}$ are 4 times $\frac{1}{3}$. Twice $\frac{1}{3}$ are twice the fourth of 4 times $\frac{1}{3}$.

$\frac{5}{3}$ are 5 times $\frac{1}{3}$. Twice $\frac{1}{3}$ are twice the fifth of 5 times $\frac{1}{3}$.

And so on.

Ratio of $\frac{3}{3}$ to any number of thirds.

$\frac{3}{3}$ are 3 times $\frac{1}{3}$.

$\frac{5}{3}$ are twice $\frac{1}{3}$. 3 times $\frac{1}{3}$ are 3 times the half of twice $\frac{1}{3}$.

$\frac{3}{3}$ are 3 times $\frac{1}{3}$. 3 times $\frac{1}{3}$ are 3 times the third of 3 times $\frac{1}{3}$.

$\frac{4}{3}$ are 4 times $\frac{1}{3}$. 3 times $\frac{1}{3}$ are 3 times the fourth of 4 times $\frac{1}{3}$.

$\frac{5}{3}$ are 5 times $\frac{1}{3}$. 3 times $\frac{1}{3}$ are 3 times the fifth of 5 times $\frac{1}{3}$.

And so on.

Ratio of $\frac{4}{3}$ to any number of thirds.

$\frac{4}{3}$ are 4 times $\frac{1}{3}$.

$\frac{2}{3}$ are twice $\frac{1}{3}$. 4 times $\frac{1}{3}$ are 4 times the half of twice $\frac{1}{3}$.
$\frac{3}{3}$ are 3 times $\frac{1}{3}$. 4 times $\frac{1}{3}$ are 4 times the third of 3 times $\frac{1}{3}$.
$\frac{4}{3}$ are 4 times $\frac{1}{3}$. 4 times $\frac{1}{3}$ are 4 times the fourth of 4 times $\frac{1}{3}$.
$\frac{5}{3}$ are 5 times $\frac{1}{3}$. 4 times $\frac{1}{3}$ are 4 times the fifth of 5 times $\frac{1}{3}$.

And so on.

Ratio of $\frac{5}{3}$ to any number of thirds.

$\frac{5}{3}$ are 5 times $\frac{1}{3}$.

$\frac{2}{3}$ are twice $\frac{1}{3}$. 5 times $\frac{1}{3}$ are 5 times the half of twice $\frac{1}{3}$.
$\frac{3}{3}$ are 3 times $\frac{1}{3}$. 5 times $\frac{1}{3}$ are 5 times the third of 3 times $\frac{1}{3}$.
$\frac{4}{3}$ are 4 times $\frac{1}{3}$. 5 times $\frac{1}{3}$ are 5 times the fourth of 4 times $\frac{1}{3}$.
$\frac{5}{3}$ are 5 times $\frac{1}{3}$. 5 times $\frac{1}{3}$ are 5 times the fifth of 5 times $\frac{1}{3}$.
$\frac{6}{3}$ are 6 times $\frac{1}{3}$. 5 times $\frac{1}{3}$ are 5 times the sixth of 6 times $\frac{1}{3}$.

And so on.

And so on to the ratio of $\frac{6}{3}$, $\frac{7}{3}$, $\frac{8}{3}$, to any given number of thirds.

Third Line, or Line of Thirds.

Ratio of $\frac{2}{3}$ to the multiples of $\frac{2}{3}$.

$\frac{4}{3}$ are twice $\frac{2}{3}$. $\frac{2}{3}$ are the half of twice $\frac{2}{3}$.
$\frac{6}{3}$ are 3 times $\frac{2}{3}$. $\frac{2}{3}$ are the third of 3 times $\frac{2}{3}$.
$\frac{8}{3}$ are 4 times $\frac{2}{3}$. $\frac{2}{3}$ are the fourth of 4 times $\frac{2}{3}$.

And so on.

Ratio of $\frac{4}{3}$ to the multiples of $\frac{2}{3}$.

$\frac{4}{3}$ are twice $\frac{2}{3}$. Twice $\frac{2}{3}$ are twice the half of twice $\frac{2}{3}$.
$\frac{6}{3}$ are 3 times $\frac{2}{3}$. Twice $\frac{2}{3}$ are twice the third of 3 times $\frac{2}{3}$.
$\frac{8}{3}$ are 4 times $\frac{2}{3}$. Twice $\frac{2}{3}$ are twice the fourth of 4 times $\frac{2}{3}$.
$\frac{10}{3}$ are 5 times $\frac{2}{3}$. Twice $\frac{2}{3}$ are twice the fifth of 5 times $\frac{2}{3}$.

And so on.

Ratio of $\frac{6}{3}$ to the multiples of $\frac{2}{3}$.

$\frac{6}{3}$ are 3 times $\frac{2}{3}$. 3 times $\frac{2}{3}$ are 3 times the third of 3 times $\frac{2}{3}$

$\frac{8}{3}$ are 4 times $\frac{2}{3}$. 3 times $\frac{2}{3}$ are 3 times the fourth of 4 times $\frac{2}{3}$.

$\frac{10}{3}$ are 5 times $\frac{2}{3}$. 3 times $\frac{2}{3}$ are 3 times the fifth of 5 times $\frac{2}{3}$.

$\frac{12}{3}$ are 6 times $\frac{2}{3}$. 3 times $\frac{2}{3}$ are 3 times the sixth of 6 times $\frac{2}{3}$.

And so on.

Ratio of $\frac{8}{3}$ to the multiples of $\frac{4}{3}$.

$\frac{8}{3}$ are twice $\frac{4}{3}$. Twice $\frac{4}{3}$ are twice the half of twice $\frac{4}{3}$.

$\frac{12}{3}$ are 3 times $\frac{4}{3}$. Twice $\frac{4}{3}$ are twice the third of 3 times $\frac{4}{3}$.

$\frac{16}{3}$ are 4 times $\frac{4}{3}$. Twice $\frac{4}{3}$ are twice the fourth of 4 times $\frac{4}{3}$.

$\frac{20}{3}$ are 5 times $\frac{4}{3}$. Twice $\frac{4}{3}$ are twice the fifth of 5 times $\frac{4}{3}$.

And so on.

Ratio of $\frac{20}{3}$ to any multiples of $\frac{5}{3}$.

$\frac{20}{3}$ are 4 times $\frac{5}{3}$.

$\frac{10}{3}$ are twice $\frac{5}{3}$. 4 times $\frac{5}{3}$ are 4 times the half of twice $\frac{5}{3}$.

$\frac{15}{3}$ are 3 times $\frac{5}{3}$. 4 times $\frac{5}{3}$ are 4 times the third of 3 times $\frac{5}{3}$.

$\frac{20}{3}$ are 4 times $\frac{5}{3}$. 4 times $\frac{5}{3}$ are 4 times the fourth of 4 times $\frac{5}{3}$.

And so on.

Proceeding in this way the ratio of any multiples of $\frac{7}{3}$, $\frac{8}{3}$, $\frac{10}{3}$, &c., is determined.

Fourth Line, or Line of Fourths.

Ratio of $\frac{1}{4}$ to any number of fourths.

$\frac{1}{4}$ is the half of twice $\frac{1}{4}$ or $\frac{2}{4}$.

$\frac{1}{4}$ is the third of 3 times $\frac{1}{4}$ or $\frac{3}{4}$.

$\frac{1}{4}$ is the fourth of 4 times $\frac{1}{4}$ or $\frac{4}{4}$.

$\frac{1}{4}$ is the fifth of 5 times $\frac{1}{4}$ or $\frac{5}{4}$.

And so on.

Ratio of $\frac{2}{4}$ to any number of fourths.

$\frac{2}{4}$ are twice $\frac{1}{4}$.

$\frac{3}{4}$ are 3 times $\frac{1}{4}$. Twice $\frac{1}{4}$ are twice the third of 3 times $\frac{1}{4}$.

$\frac{4}{4}$ are 4 times $\frac{1}{4}$. Twice $\frac{1}{4}$ are twice the fourth of 4 times $\frac{1}{4}$.

$\frac{5}{4}$ are 5 times $\frac{1}{4}$. Twice $\frac{1}{4}$ are twice the fifth of 5 times $\frac{1}{4}$.

$\frac{6}{4}$ are 6 times $\frac{1}{4}$. Twice $\frac{1}{4}$ are twice the sixth of 6 times $\frac{1}{4}$.

And so on.

Ratio of $\frac{3}{4}$ to any number of fourths.

$\frac{3}{4}$ are 3 times $\frac{1}{4}$.

$\frac{2}{4}$ are twice $\frac{1}{4}$.	3 times $\frac{1}{4}$ are 3 times the half of twice $\frac{1}{4}$.
$\frac{3}{4}$ are 3 times $\frac{1}{4}$.	3 times $\frac{1}{4}$ are 3 times the third of 3 times $\frac{1}{4}$.
$\frac{4}{4}$ are 4 times $\frac{1}{4}$.	3 times $\frac{1}{4}$ are 3 times the fourth of 4 times $\frac{1}{4}$.
$\frac{5}{4}$ are 5 times $\frac{1}{4}$.	3 times $\frac{1}{4}$ are 3 times the fifth of 5 times $\frac{1}{4}$.

And so on.

Ratio of $\frac{4}{4}$ to any number of fourths.

$\frac{4}{4}$ are 4 times $\frac{1}{4}$.

$\frac{2}{4}$ are twice $\frac{1}{4}$.	4 times $\frac{1}{4}$ are 4 times the half of twice $\frac{1}{4}$.
$\frac{3}{4}$ are 3 times $\frac{1}{4}$.	4 times $\frac{1}{4}$ are 4 times the third of 3 times $\frac{1}{4}$.
$\frac{4}{4}$ are 4 times $\frac{1}{4}$.	4 times $\frac{1}{4}$ are 4 times the fourth of 4 times $\frac{1}{4}$.
are 5 times $\frac{1}{4}$.	4 times $\frac{1}{4}$ are 4 times the fifth of 5 times $\frac{1}{4}$.

And so on.

In the same manner the teacher may proceed to exhibit the ratio of $\frac{5}{4}$, $\frac{6}{4}$, $\frac{7}{4}$, &c., to any number of fourths. As in the preceding lines we have the following forms for the reduction of the ratios:

Fourth Line, or Line of Fourths.

Ratio of $\frac{3}{4}$ to the multiples of $\frac{3}{4}$.

$\frac{6}{4}$ are twice $\frac{3}{4}$.	$\frac{3}{4}$ are the half of twice $\frac{3}{4}$.
$\frac{9}{4}$ are 3 times $\frac{3}{4}$.	$\frac{3}{4}$ are the third of 3 times $\frac{3}{4}$.
$\frac{12}{4}$ are 4 times $\frac{3}{4}$.	$\frac{3}{4}$ are the fourth of 4 times $\frac{3}{4}$.

And so on.

Ratio of $\frac{6}{4}$ to the multiples of $\frac{3}{4}$.

$\frac{6}{4}$ are twice $\frac{3}{4}$.

$\frac{9}{4}$ are 3 times $\frac{3}{4}$.	Twice $\frac{3}{4}$ are twice the third of 3 times $\frac{3}{4}$.
$\frac{12}{4}$ are 4 times $\frac{3}{4}$.	Twice $\frac{3}{4}$ are twice the fourth of 4 times $\frac{3}{4}$.
$\frac{15}{4}$ are 5 times $\frac{3}{4}$.	Twice $\frac{3}{4}$ are twice the fifth of 5 times $\frac{3}{4}$.

And so on.

And so on to the ratio of $\frac{9}{4}$, $\frac{12}{4}$, $\frac{15}{4}$, &c., to the multiples of $\frac{3}{4}$. As a general illustration of the method the ratio of $\frac{15}{4}$ to the multiples of $\frac{5}{4}$ may be given.

$\frac{15}{4}$ are 3 times $\frac{5}{4}$.

$\frac{10}{4}$ are twice $\frac{5}{4}$. 3 times $\frac{5}{4}$ are 3 times the third of 3 times $\frac{5}{4}$.
$\frac{20}{4}$ are 4 times $\frac{5}{4}$. 3 times $\frac{5}{4}$ are 3 times the fourth of 4 times $\frac{5}{4}$.
$\frac{25}{4}$ are 5 times $\frac{5}{4}$. 3 times $\frac{5}{4}$ are 3 times the fifth of 5 times $\frac{5}{4}$.
$\frac{30}{4}$ are 6 times $\frac{5}{4}$. 3 times $\frac{5}{4}$ are 3 times the sixth of 6 times $\frac{5}{4}$.

And so on.

Questions on Case I.

On the Board of Simple Fractions.

Second Line, or Line of Halves.

i. What is the ratio of $\frac{1}{2}$ to $\frac{5}{2}$?....*Ans.* $\frac{1}{2}$ is the fifth of $\frac{5}{2}$.

ii. What is the ratio of $\frac{3}{2}$ to $\frac{5}{2}$?....*Ans.* 3 times the fifth.

iii. What part of 1 is $1\frac{1}{2}$?....*Ans.* 3 times the half.

Proof. $1\frac{1}{2}$ are $\frac{3}{2}$, 1 is $\frac{2}{2}$, $\frac{3}{2}$ are 3 times $\frac{1}{2}$, $\frac{2}{2}$ are twice $\frac{1}{2}$, 3 times $\frac{1}{2}$ are 3 times the half of twice $\frac{1}{2}$.

Third Line, or Line of Thirds.

i. What is the ratio of $\frac{1}{3}$ to $\frac{6}{3}$?....*Ans.* $\frac{1}{3}$ is the sixth of $\frac{6}{3}$.

ii. What part of $\frac{5}{3}$ is $\frac{2}{3}$?....*Ans.* Twice the fifth.

Proof. $\frac{5}{3}$ are 5 times $\frac{1}{3}$, $\frac{2}{3}$ are twice $\frac{1}{3}$, twice $\frac{1}{3}$ are twice the fifth of 5 times $\frac{1}{3}$.

iii. What is the ratio of $\frac{8}{3}$ to $\frac{4}{3}$?....*Ans.* $\frac{8}{3}$ are twice $\frac{4}{3}$.

iv. How many times are $\frac{4}{3}$ contained in $5\frac{1}{3}$?....*Ans.* 4 times.

Proof. $5\frac{1}{3}$ are $\frac{16}{3}$, $\frac{16}{3}$ are 4 times $\frac{4}{3}$.

v. What is the ratio of $\frac{16}{3}$ to 8?....*Ans.* $\frac{16}{3}$ are twice the third of 8.

Proof. 8 are $\frac{24}{3}$, $\frac{24}{3}$ are 3 times $\frac{8}{3}$, $\frac{16}{3}$ are twice $\frac{8}{3}$, twice $\frac{8}{3}$ are twice the third of 3 times $\frac{8}{3}$.

vi. What is the ratio of $\frac{8}{3}$ to $\frac{12}{3}$?....*Ans.* $\frac{8}{3}$ are twice the third of $\frac{12}{3}$.

Proof. $\frac{8}{3}$ are twice $\frac{4}{3}$, $\frac{12}{3}$ are 3 times $\frac{4}{3}$, twice $\frac{4}{3}$ are twice the third of 3 times $\frac{4}{3}$.

vii. What is the ratio of $\frac{10}{3}$ to $\frac{4}{3}$?....*Ans.* $\frac{10}{3}$ are 5 times the half of $\frac{4}{3}$.

Proof. $\frac{10}{3}$ are 5 times $\frac{2}{3}$, $\frac{4}{3}$ are twice $\frac{2}{3}$, 5 times $\frac{2}{3}$ are 5 times the half of twice $\frac{2}{3}$.

Fourth Line, or Line of Fourths.

i. What is the ratio of $\frac{1}{4}$ to $\frac{5}{4}$?....*Ans.* $\frac{1}{4}$ is the fifth of $\frac{5}{4}$.

ii. What is the ratio of $\frac{5}{4}$ to $\frac{3}{4}$?....*Ans.* 5 times the third.

Proof. $\frac{5}{4}$ are 5 times $\frac{1}{4}$, $\frac{3}{4}$ are 3 times $\frac{1}{4}$, 5 times $\frac{1}{4}$ are 5 times the third of 3 times $\frac{1}{4}$.

iii. What part of $\frac{9}{4}$ is $\frac{3}{4}$?....*Ans.* $\frac{3}{4}$ is the third of $\frac{9}{4}$.

iv. What is the ratio of $1\frac{1}{2}$ to $\frac{9}{4}$?....*Ans.* Twice the third.

Proof. $1\frac{1}{2}$ are $\frac{3}{2}$ or $\frac{6}{4}$, $\frac{6}{4}$ are twice $\frac{3}{4}$, $\frac{9}{4}$ are 3 times $\frac{3}{4}$, twice $\frac{3}{4}$ are twice the third of 3 times $\frac{3}{4}$.

v. Determine the ratio of $\frac{1}{2}$ of $\frac{1}{2}$ to $3\frac{1}{4}$.....*Ans.* $\frac{1}{13}$.

Proof. $\frac{1}{2}$ of $\frac{1}{2}$ is $\frac{1}{4}$, $3\frac{1}{4}$ are $\frac{13}{4}$, $\frac{1}{4}$ is the thirteenth of $\frac{13}{4}$.

Fifth Line, or Line of Fifths.

i. What part of $\frac{3}{5}$ is $\frac{2}{5}$?....*Ans.* Twice the third.

Proof. $\frac{3}{5}$ are 3 times $\frac{1}{5}$, $\frac{2}{5}$ are twice $\frac{1}{5}$, twice $\frac{1}{5}$ are twice the third of 3 times $\frac{1}{5}$.

ii. What is the ratio of $\frac{6}{5}$ to $\frac{4}{5}$?....*Ans.* 3 times the half.

Proof. $\frac{6}{5}$ are 3 times $\frac{2}{5}$, $\frac{4}{5}$ are twice $\frac{2}{5}$, 3 times $\frac{2}{5}$ are 3 times the half of twice $\frac{2}{5}$.

iii. What is the ratio of $2\frac{1}{5}$ to $1\frac{1}{5}$?....*Ans.* 11 times the sixth.

Proof. $2\frac{1}{5}$ are $\frac{11}{5}$, $1\frac{1}{5}$ are $\frac{6}{5}$, $\frac{11}{5}$ are 11 times the sixth of $\frac{6}{5}$.

iv. What part of $3\frac{3}{5}$ is $1\frac{1}{5}$?....*Ans.* The third part.

Proof. $3\frac{3}{5}$ are $\frac{18}{5}$, $1\frac{1}{5}$ are $\frac{6}{5}$, $\frac{18}{5}$ are 3 times $\frac{6}{5}$, $\frac{6}{5}$ is the third of 3 times $\frac{6}{5}$.

Money, Weights, and Measures.

Second Line, or Line of Halves.

i. If $\frac{1}{2}$ a yard of cloth cost 4*s.*, what will $\frac{3}{2}$ yards cost? ....*Ans.* 12*s.*

Proof. $\frac{3}{2}$ are 3 times $\frac{1}{2}$, the cost of $\frac{1}{2}$ is 4*s.*; therefore the cost of $\frac{3}{2}$ will be 3 times 4*s.* or 12*s.*

ii. What will be the amount of $\frac{5}{2}$ of a gallon of beer, when 1 gallon amounts to 2*s.*?....*Ans.* 5*s.*

Proof. 1 gallon contains $\frac{2}{2}$, $\frac{5}{2}$ are 5 times the half of $\frac{2}{2}$, the cost of $\frac{2}{2}$ is 2*s.*; therefore the cost of $\frac{5}{2}$, which are 5 times the $\frac{1}{2}$ of $\frac{2}{2}$, will be 5 times the half of 2*s.*, the half of 2*s.* is 1*s.*, and 5 times the half of 2*s.* are 5 times 1*s.* or 5*s.*

iii. If $1\frac{1}{2}$ yards of lace cost 9*d.*, what must I give for $2\frac{1}{2}$ yards?....*Ans.* 1*s.* 3*d.*

Proof. $1\frac{1}{2}$ are $\frac{3}{2}$, $2\frac{1}{2}$ are $\frac{5}{2}$, $\frac{5}{2}$ are 5 times the third of $\frac{3}{2}$; then as $\frac{3}{2}$ cost 9*d.*, $\frac{5}{2}$, which are 5 times the third of $\frac{3}{2}$, will cost 5 times the third of 9*d.*; the third of 9*d.* is 3*d.*, and 5 times the third of 9*d.* are 5 times 3*d.* or 15*d.* = 1*s.* 3*d.*

Third Line, or Line of Thirds.

i. What is the value of 3 lb. of tea, when $\frac{1}{3}$ lb. cost 2*s.*?.... *Ans.* 18*s.*

Proof. 3 are $\frac{9}{3}$, $\frac{9}{3}$ are 9 times $\frac{1}{3}$; then as the cost of $\frac{1}{3}$ is 2*s.*, the cost of $\frac{9}{3}$, which are 9 times $\frac{1}{3}$, will be 9 times 2*s.* or 18*s.*

ii. What is the cost of 4lb. of sugar, when $1\frac{1}{3}$ lbs. cost 1*s.* 6*d.*?....*Ans.* 4*s.* 6*d.*

Proof. 4 are $\frac{12}{3}$, $1\frac{1}{3}$ are $\frac{4}{3}$, $\frac{12}{3}$ are 3 times $\frac{4}{3}$; then as the cost of $\frac{4}{3}$ is 1*s.* 6*d.*, the cost of $\frac{12}{3}$, which is 3 times $\frac{4}{3}$, will be 3 times 1*s.* 6*d.* or 4*s.* 6*d.*

iii. What is the cost of $2\frac{2}{3}$ yards of cloth, when $1\frac{1}{3}$ yards cost 13*s.*?....*Ans.* 1*l.* 6*s.*

Proof. $2\frac{2}{3}$ are $\frac{8}{3}$, $1\frac{1}{3} = \frac{4}{3}$, $\frac{8}{3}$ are twice $\frac{4}{3}$; then as the cost of $\frac{4}{3}$ is 13*s.*, the cost of $\frac{8}{3}$, which is twice $\frac{4}{3}$, will be twice 13*s.* or 26*s.* = 1*l.* 6*s.*

Fourth Line, or Line of Fourths.

i. If 1 yard of cloth cost 12*s.*, what is the cost of $\frac{3}{4}$ yards? ....*Ans.* 9*s.*

Proof. 1 is $\frac{4}{4}$, $\frac{3}{4}$ are 3 times the fourth of $\frac{4}{4}$, the cost of 1 is 12*s.*; therefore the cost of $\frac{3}{4}$, which are 3 times the fourth of 1, will be 3 times the fourth of 12*s.*, the fourth of 12*s.* is 3*s.*, and 3 times the fourth of 12*s.* are 3 times 3*s.* or 9*s.*

ii. If 1 Flemish ell of cloth cost 15*s.*, what must be given for 1 English ell?....*Ans.* 1*l.* 5*s.*

Proof. 1 Flemish ell contains $\frac{3}{4}$ yard, 1 English ell $\frac{5}{4}$ yard; $\frac{5}{4}$ are 5 times the third of $\frac{3}{4}$; then as the cost of $\frac{3}{4}$ is 15*s.*, the cost of $\frac{5}{4}$, which are 5 times the third of $\frac{3}{4}$, will be 5 times the third of 15*s.*; the third of 15*s.* is 5*s.*, and 5 times the third of 15*s.* are 5 times 5, or 25*s.* = 1*l.* 5*s.*

iii. If $2\frac{1}{4}$ lbs. of sugar cost 2*s.* 3*d.*, what is the cost of 1 lb.? ....*Ans.* 1*s.*

Proof. $2\frac{1}{4}$ are $\frac{9}{4}$, 1 is $\frac{4}{4}$, $\frac{4}{4}$ are 4 times the ninth of $\frac{9}{4}$; then the cost of $\frac{9}{4}$ being 2*s.* 3*d.*, the cost of $\frac{4}{4}$, which is 4 times the ninth of $\frac{9}{4}$, will be 4 times the ninth of 2*s.* 3*d.*, 4 times the ninth of 27*d.* are 4 times 3*d.* or 12*d.* = 1*s.*

iv. If the $\frac{2}{3}$ of $2\frac{1}{4}$ lbs. of tobacco cost 3*s.* 9*d.*, what is the value of 5 lbs.?....*Ans.* 12*s.* 6*d.*

Proof. $2\frac{1}{4}$ are $\frac{9}{4}$, twice the third of $\frac{9}{4}$ are twice $\frac{3}{4}$ or $\frac{6}{4}$, 5 are $\frac{20}{4}$; $\frac{20}{4}$ are 10 times $\frac{2}{4}$, $\frac{6}{4}$ are 3 times $\frac{2}{4}$, 10 times $\frac{2}{4}$ are 10 times the third of 3 times $\frac{2}{4}$; then as the cost of $\frac{6}{4}$ is 3*s.* 9*d.*, the cost of $\frac{20}{4}$, which are 10 times the third of $\frac{6}{4}$, will be 10 times the third of 3*s.* 9*d.*, the third of 3*s.* 9*d.* is 1*s.* 3*d.* and 10 times the third of 3*s.* 9*d.* are 10 times 1*s.* 3*d.* or 12*s.* 6*d.*

In the preceding exercise it has been assumed that the fractions, whose ratio is to be determined, are given in the same denominator, or that they have been brought into that form by the process given in the fifth exercise. Although, for all practical purposes, this may be deemed sufficient, it will still be found instructive, in some cases, to give the entire process of demonstration, as in the following table:

Case I., continued.

On the Board of Compound Fractions.

Third Square, Fourth Line.

Ratio of $\frac{1}{4}$ to any given number of thirds.

$\frac{1}{4}$ is 3 times $\frac{1}{12}$.

$\frac{1}{3}$ is 4 times $\frac{1}{12}$. 3 times $\frac{1}{12}$ are 3 times the fourth of 4 times $\frac{1}{12}$.
$\frac{2}{3}$ are 8 times $\frac{1}{12}$. 3 times $\frac{1}{12}$ are 3 times the eighth of 8 times $\frac{1}{12}$.
$\frac{4}{3}$ are 16 times $\frac{1}{12}$. 3 times $\frac{1}{12}$ are 3 times the sixteenth of 16 times $\frac{1}{12}$.

And so on.

Ratio of $\frac{2}{4}$ to any given number of thirds.

$\frac{2}{4}$ are 6 times $\frac{1}{12}$.

$\frac{1}{3}$ is 4 times $\frac{1}{12}$. 6 times $\frac{1}{12}$ are 6 times the fourth of 4 times $\frac{1}{12}$.
$\frac{2}{3}$ are 8 times $\frac{1}{12}$. 6 times $\frac{1}{12}$ are 6 times the eighth of 8 times $\frac{1}{12}$.

And so on.

Ratio of $\frac{3}{4}$ to any given number of thirds.

$\frac{3}{4}$ are 9 times $\frac{1}{12}$.

$\frac{1}{3}$ is 4 times $\frac{1}{12}$. 9 times $\frac{1}{12}$ are 9 times the fourth of 4 times $\frac{1}{12}$.
$\frac{2}{3}$ are 8 times $\frac{1}{12}$. 9 times $\frac{1}{12}$ are 9 times the eighth of 8 times $\frac{1}{12}$.

And so on.

And so on to the ratio of $\frac{4}{4}$, $\frac{5}{4}$, $\frac{6}{4}$, $\frac{7}{4}$, &c.

On this square also the ratio of thirds to fourths may be exhibited.

Fifth Square, Second Line.

Ratio of $\frac{1}{2}$ to any given number of fifths.

$\frac{1}{2}$ is 5 times $\frac{1}{10}$.

$\frac{1}{5}$ is twice $\frac{1}{10}$. 5 times $\frac{1}{10}$ are 5 times the half of twice $\frac{1}{10}$.
$\frac{2}{5}$ are 4 times $\frac{1}{10}$. 5 times $\frac{1}{10}$ are 5 times the fourth of 4 times $\frac{1}{10}$.

And so on.

Ratio of $\frac{2}{2}$ to any given number of fifths.

$\frac{2}{2}$ are 10 times $\frac{1}{10}$.

$\frac{1}{5}$ is twice $\frac{1}{10}$. 10 times $\frac{1}{10}$ are 10 times the half of twice $\frac{1}{10}$.

$\frac{2}{5}$ are 4 times $\frac{1}{10}$. 10 times $\frac{1}{10}$ are 10 times the fourth of 4 times $\frac{1}{10}$.

And so on.

Ratio of $\frac{3}{2}$ to any given number of fifths.

$\frac{3}{2}$ are 15 times $\frac{1}{10}$.

$\frac{1}{5}$ is twice $\frac{1}{10}$. 15 times $\frac{1}{10}$ are 15 times the half of twice $\frac{1}{10}$.

$\frac{2}{5}$ are 4 times $\frac{1}{10}$. 15 times $\frac{1}{10}$ are 15 times the fourth of 4 times $\frac{1}{10}$.

$\frac{3}{5}$ are 6 times $\frac{1}{10}$. 15 times $\frac{1}{10}$ are 15 times the sixth of 6 times $\frac{1}{10}$.

And so on.

And so on to the ratio of $\frac{4}{2}$, $\frac{5}{2}$, $\frac{6}{2}$, $\frac{7}{2}$, &c.

The ratio of fifths to halves may also be exhibited on this square.

In like manner the teacher may proceed with any other square upon the board.

Questions on Case I., continued.

On the Board of Compound Fractions.

Third Square, Fourth Line.

i. What is the ratio between $\frac{1}{4}$ and $\frac{1}{3}$?....*Ans.* $\frac{1}{4}$ is 3 times the fourth of $\frac{1}{3}$.

Proof. $\frac{1}{4}$ is 3 times $\frac{1}{12}$, $\frac{1}{3}$ is 4 times $\frac{1}{12}$, 3 times $\frac{1}{12}$ are 3 times the fourth of 4 times $\frac{1}{12}$.

ii. Compare $\frac{2}{3}$ with $\frac{3}{4}$.....*Ans.* $\frac{2}{3}$ are 8 times the ninth of $\frac{3}{4}$.

Proof. $\frac{2}{3}$ are 8 times $\frac{1}{12}$, $\frac{3}{4}$ are 9 times $\frac{1}{12}$, 8 times $\frac{1}{12}$ are 8 times the ninth of 9 times $\frac{1}{12}$.

iii. $\frac{5}{4}$ are how many times $\frac{2}{3}$?....*Ans.* 15 times the eighth.

Proof. $\frac{5}{4}$ are 15 times $\frac{1}{12}$, $\frac{2}{3}$ are 8 times $\frac{1}{12}$, 15 times $\frac{1}{12}$ are 15 times the eighth of 8 times $\frac{1}{12}$.

Second Square, Fifth Line.

i. How many fifths are there contained in $\frac{3}{2}$?....*Ans.* $7\frac{1}{2}$ fifths.

Proof. $\frac{1}{5}$ is twice $\frac{1}{10}$, $\frac{3}{2}$ are 15 times $\frac{1}{10}$, 15 times $\frac{1}{10}$ are 15 times the half of twice $\frac{1}{10}$.

ii. What part of $\frac{1}{2}$ is $\frac{4}{5}$?....*Ans.* $\frac{4}{5}$ are 8 times the fifth of $\frac{1}{2}$.

Proof. $\frac{1}{2}$ is 5 times $\frac{1}{10}$, $\frac{4}{5}$ are 8 times $\frac{1}{10}$, 8 times $\frac{1}{10}$ are 8 times the fifth of 5 times $\frac{1}{10}$.

iii. $2\frac{1}{2}$ are how many times $\frac{2}{5}$?....*Ans.* 25 times the fourth.

Proof. $2\frac{1}{2}$ are 25 times $\frac{1}{10}$, $\frac{2}{5}$ are 4 times $\frac{1}{10}$, 25 times $\frac{1}{10}$ are 25 times the fourth of 4 times $\frac{1}{10}$.

And so on to other squares.

Miscellaneous Questions.

i. What is the ratio of $\frac{2}{3}$ to $\frac{7}{3}$?....*Ans.* Twice the seventh.

ii. What is the ratio of $\frac{3}{4}$ to $1\frac{1}{2}$?....*Ans.* $\frac{3}{4}$ are the half of $1\frac{1}{2}$.

iii. What part of $\frac{3}{4}$ is $\frac{1}{4}$?....*Ans.* The third.

iv. What is the ratio of $\frac{6}{5}$ to $\frac{7}{5}$?....*Ans.* 6 times the seventh.

v. Determine the ratio of $\frac{2}{3}$ of $\frac{6}{5}$ to $\frac{7}{5}$.....*Ans.* 4 times the seventh.

Proof. Twice the third of $\frac{6}{5}$ are twice $\frac{2}{5}$ or $\frac{4}{5}$, $\frac{4}{5}$ are 4 times $\frac{1}{5}$, $\frac{7}{5}$ are 7 times $\frac{1}{5}$, 4 times $\frac{1}{5}$ are 4 times the seventh of 7 times $\frac{1}{5}$.

vi. What part of $\frac{7}{6}$ is the $\frac{1}{2}$ of $\frac{2}{3}$?....*Ans.* Twice the seventh.

Proof. $\frac{1}{2}$ of $\frac{2}{3}$ are $\frac{2}{6}$, $\frac{2}{6}$ are twice the seventh of $\frac{7}{6}$.

vii. Compare $\frac{2}{3}$ and $\frac{8}{3}$.....*Ans.* $\frac{2}{3}$ are the fourth of $\frac{8}{3}$.

viii. What is the ratio of $\frac{1}{2}$ to $2\frac{1}{4}$?....*Ans.* Twice the ninth.

Proof. $\frac{1}{2}$ is $\frac{2}{4}$, $2\frac{1}{4}$ are $\frac{9}{4}$, $\frac{2}{4}$ are twice the ninth of $\frac{9}{4}$.

ix. What is the ratio of $\frac{1}{2}$ of $\frac{1}{3}$ to $\frac{5}{6}$?....*Ans.* $\frac{1}{5}$.

Proof. $\frac{1}{2}$ of $\frac{1}{3}$ is $\frac{1}{6}$, $\frac{1}{6}$ is the fifth of $\frac{5}{6}$.

x. Compare $\frac{1}{2}$ and $\frac{1}{6}$.....*Ans.* $\frac{1}{2}$ is 3 times $\frac{1}{6}$.

Proof. $\frac{1}{2}$ is $\frac{3}{6}$, $\frac{3}{6}$ are 3 times $\frac{1}{6}$.

xi. Compare $\frac{1}{3}$ and $\frac{1}{5}$.....*Ans.* $\frac{1}{3}$ is 5 times the third of $\frac{1}{5}$.

Proof. $\frac{1}{3}$ are $\frac{5}{15}$, $\frac{1}{5}$ is $\frac{3}{15}$, $\frac{5}{15}$ are 5 times the third of $\frac{3}{15}$.

xii. What part of $2\frac{1}{2}$ is the $\frac{1}{2}$ of $\frac{1}{2}$?....*Ans.* The tenth.

Proof. The $\frac{1}{2}$ of $\frac{1}{2}$ is $\frac{1}{4}$, $2\frac{1}{2}$ are $\frac{5}{2}$ or $\frac{10}{4}$, $\frac{1}{4}$ is the tenth of $\frac{10}{4}$.

xiii. Compare $2\frac{4}{5}$ with $1\frac{1}{5}$.....*Ans.* 7 times the third.

Proof. $2\frac{4}{5}$ are $\frac{14}{5}$, $1\frac{1}{5}$ are $\frac{6}{5}$, $\frac{14}{5}$ are 7 times $\frac{2}{5}$, $\frac{6}{5}$ are 3 times $\frac{2}{5}$, 7 times $\frac{2}{5}$ are 7 times the third of 3 times $\frac{2}{5}$.

xiv. If $\frac{3}{5}$ of a yard cost 6*s.*, what must I give for $\frac{2}{5}$ yard?....*Ans.* 4*s.*

Proof. $\frac{2}{5}$ are twice the third of $\frac{3}{5}$; then if 6*s.* are given for $\frac{3}{5}$ yards, twice the third of 6*s.* must be given for $\frac{2}{5}$; the third of 6*s.* is 2*s.*, and twice the third of 6*s.* are twice 2*s.* or 4*s.*

xv. If $\frac{1}{4}$ lb. cost $6\frac{1}{2}$*d.* what will $2\frac{1}{2}$ lbs. cost?....*Ans.* 5*s.* 5*d.*

Proof. $2\frac{1}{2}$ are $\frac{5}{2}$ or $\frac{10}{4}$; $\frac{10}{4}$ are 10 times $\frac{1}{4}$; therefore the cost of $\frac{10}{4}$ will be 10 times $6\frac{1}{2}$*d.* or 5*s.* 5*d.*

xvi. What is the cost of $2\frac{1}{8}$ ozs. of silver if $\frac{3}{4}$ of an oz. cost 3*s.* 9*d.*?....*Ans.* 10*s.* $7\frac{1}{2}$*d.*

Proof. $2\frac{1}{8}$ are $\frac{17}{8}$, $\frac{3}{4}$ are $\frac{6}{8}$, $\frac{17}{8}$ are 17 times the sixth of $\frac{6}{8}$; then if the cost of $\frac{6}{8}$ be 3*s.* 9*d.* or 45*d.*, the cost of $\frac{17}{8}$, which are 17 times the sixth of $\frac{6}{8}$, will be 17 times the sixth of 45*d.*; the sixth of 45*d.* is $7\frac{1}{2}$*d.*, and 17 times $7\frac{1}{2}$*d.* are $127\frac{1}{2}$*d.*= 10*s.* $7\frac{1}{2}$*d.*

xvii. Find the price of $4\frac{1}{4}$ lbs. of tea, when $\frac{1}{8}$ lbs. cost 7*d.*....*Ans.* 19*s.* 10*d.*

Proof. $4\frac{1}{4}$ are $\frac{17}{4}$ or $\frac{34}{8}$, $\frac{34}{8}$ are 34 times $\frac{1}{8}$; then if the cost of $\frac{1}{8}$ be 7*d.*, the cost of $\frac{34}{8}$, which are 34 times $\frac{1}{8}$, will be 34 times 7*d.* or 238*d.* = 19*s.* 10*d.*

xviii. A person possesses $\frac{3}{4}$ of $\frac{8}{9}$ of a ship, and $\frac{2}{3}$ of his share is worth 360*l.*, what is the worth of the whole ship?.... *Ans.* 810*l.*

Proof. $\frac{3}{4}$ of $\frac{8}{9}$ are $\frac{2}{3}$, and $\frac{2}{3}$ of $\frac{2}{3}$ are $\frac{4}{9}$, $\frac{4}{9}$ therefore is sold for 360*l.*; and $\frac{1}{9}$ will be worth the fourth of 360*l.* = 90*l.*; and $\frac{9}{9}$, or the whole ship, will be worth 9 times 90*l.* = 810*l.*

Case II. *When a given Fraction is a given part of a Fraction required.*

Example. Of what fraction is $\frac{2}{3}$ five times the seventh part?....*Ans.* $\frac{14}{15}$.

The principle involved in this case of ratios differs so little from that of the preceding case, that a few illustrations will suffice to exhibit the form of the exercise.

On the Board of Simple Fractions.

Line of Halves.

Ratio of $\frac{1}{2}$ to any number of halves.

$\frac{1}{2}$ is the half of twice $\frac{1}{2}$ or $\frac{2}{2}$
$\frac{1}{2}$ is the third of 3 times $\frac{1}{2}$ or $\frac{3}{2}$
$\frac{1}{2}$ is the fourth of 4 times $\frac{1}{2}$ or $\frac{4}{2}$
And so on.

Ratio of $\frac{2}{2}$ to any number of halves.

$\frac{2}{2}$ are twice $\frac{1}{2}$
Twice $\frac{1}{2}$ are twice the third of 3 times $\frac{1}{2}$ or $\frac{3}{2}$
Twice $\frac{1}{2}$ are twice the fourth of 4 times $\frac{1}{2}$ or $\frac{4}{2}$
Twice $\frac{1}{2}$ are twice the fifth of 5 times $\frac{1}{2}$ or $\frac{5}{2}$
And so on.

Ratio of $\frac{3}{2}$ to any number of halves.

$\frac{3}{2}$ are 3 times $\frac{1}{2}$
3 times $\frac{1}{2}$ are 3 times the half of twice $\frac{1}{2}$ or $\frac{2}{2}$.
3 times $\frac{1}{2}$ are 3 times the third of 3 times $\frac{1}{2}$ or $\frac{3}{2}$
3 times $\frac{1}{2}$ are 3 times the fourth of 4 times $\frac{1}{2}$ or $\frac{4}{2}$
And so on.

Then follows the ratio of $\frac{4}{2}$, $\frac{5}{2}$, $\frac{6}{2}$, &c.

Reduction of Ratios.

Line of Halves.

Ratio of $\frac{6}{2}$ to any multiples of $\frac{3}{2}$.

$\frac{6}{2}$ are twice $\frac{3}{2}$

Twice $\frac{3}{2}$ are twice the third of 3 times $\frac{3}{2}$ or $\frac{9}{2}$

Twice $\frac{3}{2}$ are twice the fourth of 4 times $\frac{3}{2}$ or $\frac{12}{2}$

Twice $\frac{3}{2}$ are twice the fifth of 5 times $\frac{3}{2}$ or $\frac{15}{2}$

And so on.

Ratio of $\frac{9}{2}$ to any multiples of $\frac{3}{2}$.

$\frac{9}{2}$ are 3 times $\frac{3}{2}$

3 times $\frac{3}{2}$ are 3 times the half of twice $\frac{3}{2}$ or $\frac{6}{2}$

3 times $\frac{3}{2}$ are 3 times the third of $\frac{3}{2}$ or $\frac{9}{2}$

3 times $\frac{3}{2}$ are 3 times the fourth of 4 times $\frac{3}{2}$ or $\frac{12}{2}$

And so on.

And so on to the ratio of $\frac{12}{2}$, $\frac{15}{2}$, $\frac{18}{2}$, &c., to any multiples of $\frac{3}{2}$.

Ratio of $\frac{10}{2}$ to any multiples of $\frac{5}{2}$.

$\frac{10}{2}$ are twice $\frac{5}{2}$

Twice $\frac{5}{2}$ are twice the third of 3 times $\frac{5}{2}$ or $\frac{15}{2}$

Twice $\frac{5}{2}$ are twice the fourth of 4 times $\frac{5}{2}$ or $\frac{20}{2}$

Twice $\frac{5}{2}$ are twice the fifth of 5 times $\frac{5}{2}$ or $\frac{25}{2}$

And so on.

Ratio of $\frac{15}{2}$ to any multiples of $\frac{5}{2}$.

$\frac{15}{2}$ are 3 times $\frac{5}{2}$

3 times $\frac{5}{2}$ are 3 times the half of twice $\frac{5}{2}$ or $\frac{10}{2}$

3 times $\frac{5}{2}$ are 3 times the third of 3 times $\frac{5}{2}$ or $\frac{15}{2}$

3 times $\frac{5}{2}$ are 3 times the fourth of 4 times $\frac{5}{2}$ or $\frac{20}{2}$

And so on.

And so on to any other ratios.

Line of Thirds.

Ratio of $\frac{1}{3}$ to any number of thirds.
$\frac{1}{3}$ is the half of twice $\frac{1}{3}$ or $\frac{2}{3}$
$\frac{1}{3}$ is the third of 3 times $\frac{1}{3}$ or $\frac{3}{3}$
$\frac{1}{3}$ is the fourth of 4 times $\frac{1}{3}$ or $\frac{4}{3}$
And so on.

Ratio of $\frac{2}{3}$ to any number of thirds.
$\frac{2}{3}$ are twice $\frac{1}{3}$
Twice $\frac{1}{3}$ are twice the third of 3 times $\frac{1}{3}$ or $\frac{3}{3}$
Twice $\frac{1}{3}$ are twice the fourth of 4 times $\frac{1}{3}$ or $\frac{4}{3}$
Twice $\frac{1}{3}$ are twice the fifth of 5 times $\frac{1}{3}$ or $\frac{5}{3}$
And so on.

Then follows the ratio of $\frac{3}{3}$, $\frac{4}{3}$, $\frac{5}{3}$, &c., to any number of thirds.

Line of Thirds.

Ratio of $\frac{4}{3}$ to any multiples of $\frac{2}{3}$.
$\frac{4}{3}$ are twice $\frac{2}{3}$
Twice $\frac{2}{3}$ are twice the third of 3 times $\frac{2}{3}$ or $\frac{6}{3}$
Twice $\frac{2}{3}$ are twice the fourth of 4 times $\frac{2}{3}$ or $\frac{8}{3}$
Twice $\frac{2}{3}$ are twice the fifth of 5 times $\frac{2}{3}$ or $\frac{10}{3}$
And so on.

Ratio of $\frac{6}{3}$ to any multiples of $\frac{2}{3}$.
$\frac{6}{3}$ are 3 times $\frac{2}{3}$
3 times $\frac{2}{3}$ are 3 times the half of twice $\frac{2}{3}$ or $\frac{4}{3}$
3 times $\frac{2}{3}$ are 3 times the third of 3 times $\frac{2}{3}$ or $\frac{6}{3}$
3 times $\frac{2}{3}$ are 3 times the fourth of 4 times $\frac{2}{3}$ or $\frac{8}{3}$
And so on.

Then follows the ratio of $\frac{8}{3}$, $\frac{10}{3}$, $\frac{12}{3}$, &c., to any multiples of $\frac{2}{3}$.

Ratio of $\frac{10}{3}$ to any multiples of $\frac{5}{3}$.

$\frac{10}{3}$ are twice $\frac{5}{3}$

Twice $\frac{5}{3}$ are twice the third of 3 times $\frac{5}{3}$ or $\frac{15}{3}$

Twice $\frac{5}{3}$ are twice the fourth of 4 times $\frac{5}{3}$ or $\frac{20}{3}$

Twice $\frac{5}{3}$ are twice the fifth of 5 times $\frac{5}{3}$ or $\frac{25}{3}$

And so on.

Ratio of $\frac{15}{3}$ to any multiples of $\frac{5}{3}$.

$\frac{15}{3}$ are 3 times $\frac{5}{3}$

3 times $\frac{5}{3}$ are 3 times the half of twice $\frac{5}{3}$ or $\frac{10}{3}$

3 times $\frac{5}{3}$ are 3 times the third of 3 times $\frac{5}{3}$ or $\frac{15}{3}$

3 times $\frac{5}{3}$ are 3 times the fourth of 4 times $\frac{5}{3}$ or $\frac{20}{3}$

And so on.

Then follows the ratio of $\frac{20}{3}$, $\frac{25}{3}$, &c., to any multiples of $\frac{5}{3}$. And so on to any other ratios.

Besides giving the ratio of fractions, this exercise furnishes a demonstrative method for effecting the *division of fractions*. When the divisor is greater than the dividend, it cannot be said, without some modification of language, that the one quantity is contained in the other. To avoid this difficulty it may, however, be said, without any impropriety, that the one is contained in the other a certain fraction of once.

The most general definition of division is, that the quotient, or result, multiplied by the divisor, is equal to the dividend. For example, the quotient of $\frac{3}{4}$ divided by $\frac{2}{5}$ must be a quantity, which multiplied by $\frac{2}{5}$ will give $\frac{3}{4}$; or $\frac{2}{5} \times$ quotient $= \frac{3}{4}$; that is, $\frac{3}{4}$ is twice the fifth of the quotient or number required; the $\frac{1}{5}$ of the quotient, therefore, will be the half of $\frac{3}{4}$, and the quotient itself will be 5 times the half of $\frac{3}{4}$, or $\frac{5}{2}$ of $\frac{3}{4}$. Hence we observe, that *to divide one fraction by another, we must invert the divisor and then proceed as in Multiplication.* For instance, to divide $\frac{14}{21}$ by $\frac{7}{8}$ is the same thing as finding a number of which $\frac{14}{21}$ is the $\frac{7}{8}$ part; the operation of which, by the exercise, is as follows: $\frac{14}{21}$ are 7 times $\frac{2}{21}$, 7 times $\frac{2}{21}$ are 7 times the eighth of 8 times $\frac{2}{21}$ or $\frac{16}{21}$.

Questions on Case II.

On the Board of Simple Fractions.

Line of Halves.

i. $\frac{3}{2}$ are 3 times the fourth of what number?....*Ans.* 2.

Proof. 3 times $\frac{1}{2}$ are 3 times the fourth of 4 times $\frac{1}{2}$ or $\frac{4}{2}$, $\frac{4}{2}$ are 2 whole numbers.

ii. $\frac{9}{2}$ are 3 times the fifth of what number?....*Ans.* $\frac{15}{2} = 7\frac{1}{2}$.

Proof. $\frac{9}{2}$ are 3 times $\frac{3}{2}$, 3 times $\frac{3}{2}$ are 3 times the fifth of 5 times $\frac{3}{2}$ or $\frac{15}{2} = 7\frac{1}{2}$.

Line of Thirds.

i. $\frac{2}{3}$ are twice the fifth of what number?....*Ans.* $\frac{5}{3} = 1\frac{2}{3}$.

Proof. $\frac{2}{3}$ are twice $\frac{1}{3}$, twice $\frac{1}{3}$ are twice the fifth of 5 times $\frac{1}{3}$ or $\frac{5}{3} = 1\frac{2}{3}$.

ii. $\frac{6}{3}$ are 3 times the fourth of what number?....*Ans.* $\frac{8}{3} = 2\frac{2}{3}$.

Proof. $\frac{6}{3}$ are 3 times $\frac{2}{3}$, 3 times $\frac{2}{3}$ are 3 times the fourth of 4 times $\frac{2}{3}$ or $\frac{8}{3} = 2\frac{2}{3}$.

iii. $\frac{15}{3}$ are 3 times the half of what number?....*Ans.* $\frac{10}{3} = 3\frac{1}{3}$.

Proof. $\frac{15}{3}$ are 3 times $\frac{5}{3}$, 3 times $\frac{5}{3}$ are 3 times the half of twice $\frac{5}{3}$ or $\frac{10}{3} = 3\frac{1}{3}$.

iv. $\frac{10}{3}$ are twice the fifth of what number?....*Ans.* $\frac{25}{3} = 8\frac{1}{3}$.

Proof. $\frac{10}{3}$ are twice $\frac{5}{3}$, twice $\frac{5}{3}$ are twice the fifth of 5 times $\frac{5}{3}$ or $\frac{25}{3} = 8\frac{1}{3}$.

Line of Fourths.

i. $\frac{1}{4}$ is the third of what number?....*Ans.* $\frac{3}{4}$.

Proof. $\frac{1}{4}$ is the third of 3 times $\frac{1}{4}$ or $\frac{3}{4}$.

ii. $\frac{3}{4}$ are 3 times the seventh of what number?....*Ans.* $\frac{7}{4} = 1\frac{3}{4}$.

Proof. $\frac{3}{4}$ are 3 times $\frac{1}{4}$, 3 times $\frac{1}{4}$ are 3 times the seventh of 7 times $\frac{1}{4}$ or $\frac{7}{4} = 1\frac{3}{4}$.

iii. $\frac{15}{4}$ are 5 times the half of what number?....*Ans.* $\frac{6}{4}=1\frac{1}{2}$.

Proof. $\frac{15}{4}$ are 5 times $\frac{3}{4}$, 5 times $\frac{3}{4}$ are 5 times the half of twice $\frac{3}{4}$ or $\frac{6}{4}=1\frac{1}{2}$.

iv. $\frac{9}{4}$ are 3 times the fifth of what number?....*Ans.* $\frac{15}{4}=3\frac{3}{4}$.

Proof. $\frac{9}{4}$ are 3 times $\frac{3}{4}$, 3 times $\frac{3}{4}$ are 3 times the fifth of 5 times $\frac{3}{4}$ or $\frac{15}{4}=3\frac{3}{4}$.

Case II., continued.

On the Board of Compound Fractions.

Third Square, Second Line.

Ratio of $\frac{1}{2}$ to any number of thirds.

$\frac{1}{2}$ is 3 times $\frac{1}{6}$

3 times $\frac{1}{6}$ are 3 times the half of twice $\frac{1}{6}$ or $\frac{1}{3}$
3 times $\frac{1}{6}$ are 3 times the fourth of 4 times $\frac{1}{6}$ or $\frac{2}{3}$
3 times $\frac{1}{6}$ are 3 times the sixth of 6 times $\frac{1}{6}$ or 1
3 times $\frac{1}{6}$ are 3 times the eighth of 8 times $\frac{1}{6}$ or $1\frac{1}{3}$

And so on.

Ratio of $\frac{2}{2}$ to any number of thirds.

$\frac{2}{2}$ are 6 times $\frac{1}{6}$

6 times $\frac{1}{6}$ are 6 times the half of twice $\frac{1}{6}$ or $\frac{1}{3}$
6 times $\frac{1}{6}$ are 6 times the fourth of 4 times $\frac{1}{6}$ or $\frac{2}{3}$
6 times $\frac{1}{6}$ are 6 times the sixth of 6 times $\frac{1}{6}$ or 1
6 times $\frac{1}{6}$ are 6 times the eighth of 8 times $\frac{1}{6}$ or $\frac{4}{3}$

And so on.

Ratio of $\frac{3}{2}$ to any number of thirds.

$\frac{3}{2}$ are 9 times $\frac{1}{6}$

9 times $\frac{1}{6}$ are 9 times the half of twice $\frac{1}{6}$ or $\frac{1}{3}$
9 times $\frac{1}{6}$ are 9 times the fourth of 4 times $\frac{1}{6}$ or $\frac{2}{3}$
9 times $\frac{1}{6}$ are 9 times the sixth of 6 times $\frac{1}{6}$ or 1

And so on.

And so on to the ratio of $\frac{4}{2}$, $\frac{5}{2}$, $\frac{6}{2}$, &c., to any number of thirds.

This square also exhibits the ratio of thirds to halves.

Any other square may be treated in a similar manner.

Questions on Case II., continued.

On the Board of Compound Fractions.

Third Square, Second Line.

i. $\frac{1}{2}$ is 3 times the fourth of what number?....*Ans.* $\frac{2}{3}$.

Proof. $\frac{1}{2}$ is 3 times $\frac{1}{6}$, 3 times $\frac{1}{6}$ are 3 times the fourth of 4 times $\frac{1}{6}$ or $\frac{2}{3}$.

ii. $\frac{3}{2}$ are 9 times the fourth of what number?....*Ans.* $\frac{2}{3}$.

Proof. $\frac{3}{2}$ are 9 times $\frac{1}{6}$, 9 times $\frac{1}{6}$ are 9 times the fourth of 4 times $\frac{1}{6}$ or $\frac{2}{3}$.

Fifth Square, Third Line.

i. $\frac{1}{5}$ is 3 times the fourth of what quantity?....*Ans.* $\frac{4}{15}$.

Proof. $\frac{1}{5}$ is 3 times $\frac{1}{15}$, 3 times $\frac{1}{15}$ are 3 times the fourth of 4 times $\frac{1}{15}$ or $\frac{4}{15}$.

ii. $\frac{2}{3}$ are 5 times the sixth of what quantity?....*Ans.* $\frac{12}{15} = \frac{4}{5}$.

Proof. $\frac{2}{3}$ are 5 times $\frac{2}{15}$, 5 times $\frac{2}{15}$ are 5 times the sixth of 6 times $\frac{2}{15}$ or $\frac{12}{15}$, $\frac{12}{15}$ are $\frac{4}{5}$.

iii. $\frac{4}{3}$ are 5 times the ninth of what quantity?....*Ans.* $2\frac{2}{5}$.

Proof. $\frac{4}{3}$ are 5 times $\frac{4}{15}$, 5 times $\frac{4}{15}$ are 5 times the ninth of 9 times $\frac{4}{15}$ or $\frac{36}{15} = 2\frac{2}{5}$.

Miscellaneous Questions.

i. Of what fraction is $\frac{1}{2}$ the third part?....*Ans.* $1\frac{1}{2}$.

Proof. $\frac{1}{2}$ is the third of 3 times $\frac{1}{2}$ or $\frac{3}{2} = 1\frac{1}{2}$.

ii. Of what is $\frac{2}{3}$ the fourth part?....*Ans.* $2\frac{2}{3}$.

Proof. $\frac{2}{3}$ is the fourth of 4 times $\frac{2}{3}$ or $\frac{8}{3} = 2\frac{2}{3}$.

iii. $\frac{5}{2}$ are 6 times what number?....*Ans.* $\frac{5}{12}$.

Proof. $\frac{5}{2}$ are 6 times the sixth of $\frac{5}{2}$, the sixth of $\frac{5}{2}$ is $\frac{5}{12}$.

iv. $\frac{3}{4}$ are 5 times what number?....*Ans.* $\frac{3}{20}$.

Proof. $\frac{3}{4}$ are 5 times the fifth of $\frac{3}{4}$, the fifth of $\frac{3}{4}$ is $\frac{3}{20}$.

v. Of what number is $\frac{3}{2}$, 4 times the third?....*Ans.* $1\frac{1}{8}$.

Proof. $\frac{3}{2}$ are 4 times $\frac{3}{8}$, 4 times $\frac{3}{8}$ are 4 times the third of 3 times $\frac{3}{8}$ or $\frac{9}{8} = 1\frac{1}{8}$.

vi. $3\frac{1}{2}$ are 4 times the seventh of what number?....*Ans.* $6\frac{1}{8}$.

Proof. $3\frac{1}{2}$ are $\frac{7}{2}$, $\frac{7}{2}$ are 4 times $\frac{7}{8}$, 4 times $\frac{7}{8}$ are 4 times the seventh of 7 times $\frac{7}{8}$ or $\frac{49}{8} = 6\frac{1}{8}$.

vii. $1\frac{1}{3}$ is 3 times the fifth of what number?....*Ans.* $2\frac{2}{9}$.

Proof. $1\frac{1}{3}$ are $\frac{4}{3}$, $\frac{4}{3}$ are 3 times $\frac{4}{9}$, 3 times $\frac{4}{9}$ are 3 times the fifth of 5 times $\frac{4}{9}$ or $\frac{20}{9} = 2\frac{2}{9}$.

viii. $2\frac{7}{8}$ is the fourth of what number?....*Ans.* $11\frac{1}{2}$.

Proof. $2\frac{7}{8}$ are $\frac{23}{8}$, $\frac{23}{8}$ the fourth of 4 times $\frac{23}{8}$ or $\frac{92}{8} = 11\frac{1}{2}$.

ix. Of what is the third of $\frac{6}{7}$, the fifth part?....*Ans.* $1\frac{3}{7}$.

Proof. The third of $\frac{6}{7}$ is $\frac{2}{7}$, $\frac{2}{7}$ is the fifth of 5 times $\frac{2}{7}$ or $\frac{10}{7} = 1\frac{3}{7}$.

x. Of what number is the $\frac{7}{8}$ of 3 the half?....*Ans.* $5\frac{1}{4}$.

Proof. The eighth of 3 is $\frac{3}{8}$, and 7 times the eighth of 3 are 7 times $\frac{3}{8}$ or $\frac{21}{8}$, $\frac{21}{8}$ is the half of twice $\frac{21}{8}$ or $\frac{42}{8} = 5\frac{1}{4}$.

xi. $7\frac{1}{2}$ are 5 times what number?....*Ans.* $1\frac{1}{2}$.

Proof. $7\frac{1}{2}$ are $\frac{15}{2}$, $\frac{15}{2}$ are 5 times the fifth of $\frac{15}{2}$, the fifth of $\frac{15}{2}$ is $\frac{3}{2} = 1\frac{1}{2}$.

xii. 5 times the sixth of $3\frac{1}{2}$ is 6 times what number?.... *Ans.* $\frac{35}{72}$.

Proof. $3\frac{1}{2}$ are $\frac{7}{2}$, the sixth of $\frac{7}{2}$ is $\frac{7}{12}$, and 5 times the sixth of $\frac{7}{2}$ are 5 times $\frac{7}{12}$ or $\frac{35}{12}$, $\frac{35}{12}$ are 6 times the sixth of $\frac{35}{12}$, the sixth of $\frac{35}{12}$ is $\frac{35}{72}$.

xiii. Of what number is $\frac{1}{2}$ of $\frac{3}{4}$, 3 times the eighth part? ....*Ans.* 1.

Proof. $\frac{1}{2}$ of $\frac{3}{4}$ is $\frac{3}{8}$, $\frac{3}{8}$ are 3 times $\frac{1}{8}$, 3 times $\frac{1}{8}$ are 3 times the eighth of 8 times $\frac{1}{8}$ or 1.

xiv. The third and fifth of a number is 24; required the number.....*Ans.* 45.

Proof. $\frac{1}{3}$ and $\frac{1}{5}$ are $\frac{8}{15}$, that is, $\frac{8}{15}$ of the number required is 24; then 24 are 8 times 3, 8 times 3 are 8 times the fifteenth of 15 times 3 or 45.

xv. $\frac{3}{4}$ of my property sold for 270*l.*, required the value of the whole.....*Ans.* 360*l.*

Proof. 270 are 3 times 90, 3 times 90 are 3 times the fourth of 4 times 90 or 360.

xvi. What is the worth of a field, when $\frac{3}{5}$ of $\frac{2}{3}$ sold for 180*l.*?....*Ans.* 450*l.*

Proof. $\frac{3}{5}$ of $\frac{2}{3} = \frac{2}{5}$; then 180 are twice 90, twice 90 are twice the fifth of 5 times 90 or 450. Or thus; as $\frac{2}{5}$ is worth 180*l.*, $\frac{1}{5}$ will be worth the half of 180*l.*, that is, 90*l.*; and 1 or $\frac{5}{5}$ will be 5 times 90 or 450*l.*

xvii. 2 are 3 times $\frac{1}{5}$ of what number?....*Ans.* $\frac{10}{3} = 3\frac{1}{3}$.

Proof. 2 are 3 times $\frac{2}{3}$; 3 times $\frac{2}{3}$ are 3 times $\frac{1}{5}$ of 5 times $\frac{2}{3}$ or $\frac{10}{3}$.

xviii. 5*s.* are 3 times $\frac{1}{4}$ of what sum?....*Ans.* 6*s.* 8*d.*

Proof. 5 are 3 times $\frac{5}{3}$; 3 times $\frac{5}{3}$ are 3 times $\frac{1}{4}$ of 4 times $\frac{5}{3}$ or $\frac{20}{3}$, $\frac{20}{3}$*s.* $= 6\frac{2}{3}$*s.* $=$ 6*s.* 8*d.*

xix. $\frac{7}{5}$ of a number is 4; what is the number?....*Ans.* $\frac{20}{7} = 2\frac{6}{7}$.

Proof. 4 are 7 times $\frac{4}{7}$; 7 times $\frac{4}{7}$ are 7 times $\frac{1}{5}$ of 5 times $\frac{4}{7}$ or $\frac{20}{7} = 2\frac{6}{7}$. Or thus; as 7 times $\frac{1}{5}$ of the number required $= 4$; $\frac{1}{5}$ of the number will be the $\frac{1}{7}$ of 4 or $\frac{4}{7}$; and therefore the number itself $=$ 5 times $\frac{4}{7}$ or $\frac{20}{7}$.

ADDENDA TO THE EIGHTH EXERCISE.

Division of Fractions.

1. Division of Whole Numbers by a Fraction.

i. Divide 1 by $\frac{1}{4}$.....*Ans.* 4.

Proof. Because $1 = \frac{4}{4}$; therefore $\frac{4}{4} \div \frac{1}{4} = 4$.

ii. Divide 2 by $\frac{1}{3}$.....*Ans.* 6.

Proof. $2 = \frac{6}{3}$; therefore $\frac{6}{3} \div \frac{1}{3} = 6$.

iii. Divide 1 by $\frac{2}{3}$.....*Ans.* $\frac{3}{2} = 1\frac{1}{2}$.

Proof. $1 = \frac{3}{3}$, and $\frac{3}{3} \div \frac{1}{3} = 3$; therefore $\frac{3}{3} \div \frac{2}{3} =$ the half of 3, that is, $\frac{3}{2}$.

iv. Divide 4 by $\frac{7}{3}$.....*Ans.* $\frac{12}{7} = 1\frac{5}{7}$.

Proof. $4 = \frac{12}{3}$, and $\frac{12}{3} \div \frac{1}{3} = 12$; therefore $\frac{12}{3} \div \frac{7}{3} =$ the seventh of 12, that is, $\frac{12}{7}$.

v. Divide 7 by $\frac{1}{3}$.....*Ans.* 21.

Proof. Because each unit contains $\frac{1}{3}$ three times; therefore 7 will contain seven times 3 or 21 thirds.

vi. How many halves are there contained in 5?....*Ans.* 10. Because each unit contains 2 halves, and therefore 5 will contain 5 times 2 or 10 halves.

vii. How often will $\frac{3}{2}$, then, be contained in 5?....*Ans.* The third of 10 times, or $\frac{10}{3}$. Because $\frac{3}{2}$ will be contained the third of the number of times which $\frac{1}{2}$ is contained in the given fraction.

viii. Divide 5 by $\frac{5}{9}$.....*Ans.* 9.

ix. Divide 3 by $7\frac{1}{2}$.....*Ans.* $\frac{6}{15}=\frac{2}{5}$.

x. Divide 9 by $\frac{2}{3}$.....*Ans.* $\frac{27}{2}=13\frac{1}{2}$.

2. Division of a Fraction by a Whole Number.

i. Divide $\frac{1}{3}$ by 1.....*Ans.* $\frac{1}{3}$.

Because $1=\frac{3}{3}$, and $\frac{1}{3}\div\frac{1}{3}=1$; therefore $\frac{1}{3}\div\frac{3}{3}=$ the third of 1 or $\frac{1}{3}$.

ii. Divide $\frac{1}{3}$ by 4.....*Ans.* $\frac{1}{12}$.

Proof. $\frac{1}{3}\div\frac{1}{3}=1$, but $4=\frac{12}{3}$; therefore $\frac{1}{3}\div\frac{12}{3}=$ the $\frac{1}{12}$ of 1, that is, $\frac{1}{12}$.

iii. Divide $\frac{1}{4}$ by 5, by another method.....*Ans.* $\frac{1}{20}$.

Proof. Because the quotient $\times\ 5=\frac{1}{4}$; therefore the quotient $=\frac{1}{5}$ of $\frac{1}{4}$, that is, $\frac{1}{20}$.

iv. Divide $\frac{3}{4}$ by 7.....*Ans.* $\frac{3}{28}$.

Because $\frac{3}{4}\div\frac{1}{4}=3$, but $7=\frac{28}{4}$; therefore $\frac{3}{4}\div\frac{28}{4}=$ the twenty-eighth part of 3, that is $\frac{3}{28}$.

3. Division of a Fraction by a Fraction.

i. Divide $\frac{1}{2}$ by $\frac{1}{4}$.....*Ans.* 2.

Proof. Because $\frac{1}{2}=\frac{2}{4}$, and therefore $\frac{2}{4}\div\frac{1}{4}=2$.

ii. Divide $\frac{2}{3}$ by $\frac{1}{5}$.....*Ans.* $\frac{10}{3}=3\frac{1}{3}$.

Proof. Reducing the fractions to the same denominator, $\frac{2}{3}=\frac{10}{15}$, $\frac{1}{5}=\frac{3}{15}$, then $\frac{10}{15}\div\frac{1}{15}=10$; therefore $\frac{10}{15}\div\frac{3}{15}=$ the third of 10, or $\frac{10}{3}$.

iii. Divide $\frac{3}{5}$ by $\frac{1}{4}$, by another method.....*Ans.* $\frac{12}{5}=2\frac{2}{5}$.

Proof. $\frac{1}{4}\times$ quotient $=\frac{3}{5}$; therefore quotient $=4$ times $\frac{3}{5}=\frac{12}{5}$.

iv. Divide $\frac{2}{3}$ by $\frac{7}{5}$.....*Ans.* $\frac{10}{21}$.

Proof. Reducing the fractions to the same denominator, $\frac{2}{3}=\frac{10}{15}$, $\frac{7}{5}=\frac{21}{15}$. But $\frac{1}{15}$ is contained in $\frac{10}{15}$, ten times; therefore $\frac{10}{15}\div\frac{21}{15}=$ the twenty-first part of 10, that is, $\frac{10}{21}$.

NINTH EXERCISE.

Proportion of Fractional Numbers.

This exercise is intended to show that the relations of unity, demonstrated in the Sixth Exercise of Book I. are also applicable to fractional units or fractional parts having the same name. After what has been proved in that exercise, a few illustrations, in each case, will render this truth sufficiently obvious.

This exercise comprises three cases: 1st. When the second term of the proportion is divisible by the first; 2nd. When the first term is divisible by the second; and 3rd. When the first and second terms have any ratio. The Board of Simple Fractions only is used for the exercises, but a few questions appended to the third case are given in connection with the Board of Compound Fractions.

Case I. *When the second term of the proportion is divisible by the first.*

On the Board of Simple Fractions.

Third Line, or Line of Thirds.

$\frac{2}{3}$ are twice $\frac{1}{3}$

$\frac{1}{3}$ is to $2\times\frac{1}{3}$ or $\frac{2}{3}$ as $\frac{2}{3}$ is to $2\times\frac{2}{3}$ or $\frac{4}{3}$

$\frac{4}{3}$ are twice $\frac{2}{3}$

$\frac{2}{3}:2\times\frac{2}{3}$ or $\frac{4}{3}::\frac{3}{3}:2\times\frac{3}{3}$ or $\frac{6}{3}$

$\frac{6}{3}$ are twice $\frac{3}{3}$

$\frac{3}{3}:2\times\frac{3}{3}$ or $\frac{6}{3}::\frac{4}{3}:2\times\frac{4}{3}$ or $\frac{8}{3}$.

And so on.

$\frac{3}{3}$ are $3 \times \frac{1}{3}$

$\frac{1}{3} : 3 \times \frac{1}{3}$ or $\frac{3}{3} :: \frac{2}{3} : 3 \times \frac{2}{3}$ or $\frac{6}{3}$

$\frac{6}{3}$ are $3 \times \frac{2}{3}$

$\frac{2}{3} : 3 \times \frac{2}{3}$ or $\frac{6}{3} :: \frac{3}{3} : 3 \times \frac{3}{3}$ or $\frac{9}{3}$

$\frac{9}{3}$ are $3 \times \frac{3}{3}$

$\frac{3}{3} : 3 \times \frac{3}{3}$ or $\frac{9}{3} :: \frac{4}{3} : 3 \times \frac{4}{3}$ or $\frac{12}{3}$.

And so on.

Then follow the tables beginning with $\frac{4}{3}$ are $4 \times \frac{1}{3}$, $\frac{5}{3}$ are $5 \times \frac{1}{3}$, &c.

Fifth Line, or Line of Fifths.

$\frac{2}{5}$ are $2 \times \frac{1}{5}$

$\frac{1}{5} : 2 \times \frac{1}{5}$ or $\frac{2}{5} :: \frac{3}{5} : 2 \times \frac{3}{5}$ or $\frac{6}{5}$

$\frac{4}{5}$ are $2 \times \frac{2}{5}$

$\frac{2}{5} : 2 \times \frac{2}{5}$ or $\frac{4}{5} :: \frac{5}{5} : 2 \times \frac{5}{5}$ or $\frac{10}{5}$

$\frac{6}{5}$ are $2 \times \frac{3}{5}$

$\frac{3}{5} : 2 \times \frac{3}{5}$ or $\frac{6}{5} :: \frac{7}{5} : 2 \times \frac{7}{5}$ or $\frac{14}{5}$.

And so on.

$\frac{3}{5}$ are $3 \times \frac{1}{5}$

$\frac{1}{5} : 3 \times \frac{1}{5}$ or $\frac{3}{5} :: \frac{4}{5} : 3 \times \frac{4}{5}$ or $\frac{12}{5}$

$\frac{6}{5}$ are $3 \times \frac{2}{5}$

$\frac{2}{5} : 3 \times \frac{2}{5}$ or $\frac{6}{5} :: \frac{7}{5} : 3 \times \frac{7}{5}$ or $\frac{21}{5}$

$\frac{9}{5}$ are $3 \times \frac{3}{5}$

$\frac{3}{5} : 3 \times \frac{3}{5}$ or $\frac{9}{5} :: \frac{10}{5} \cdot 3 \times \frac{10}{5}$ or $\frac{30}{5}$.

And so on.

Then follow the tables beginning with $\frac{4}{5}$ are $4 \times \frac{1}{5}$, $\frac{5}{5}$ are $5 \times \frac{1}{5}$, &c.

Similar tables may be formed for the proportion of halves, fourths, sixths, &c.

Second and Fourth Lines, or Lines of Halves and Fourths.

$\frac{2}{2}$ are twice $\frac{1}{2}$

$\frac{1}{2}$ is to twice $\frac{1}{2}$ or $\frac{2}{2}::\frac{1}{4}$:twice $\frac{1}{4}$ or

$\frac{4}{2}$ are twice $\frac{2}{2}$

$\frac{2}{2}$: twice $\frac{2}{2}$ or $\frac{4}{2}::\frac{2}{4}$: twice $\frac{2}{4}$ or $\frac{4}{4}$

$\frac{6}{2}$ are twice $\frac{3}{2}$

$\frac{3}{2}$: twice $\frac{3}{2}$ or $\frac{6}{2}::\frac{3}{4}$: twice $\frac{3}{4}$ or $\frac{6}{4}$.

And so on.

$\frac{3}{2}$ are 3 times $\frac{1}{2}$

$\frac{1}{2}$:3 times $\frac{1}{2}$ or $\frac{3}{2}::\frac{1}{4}$:3 times $\frac{1}{4}$ or $\frac{3}{4}$

$\frac{6}{2}$ are 3 times $\frac{2}{2}$

$\frac{2}{2}:3\times\frac{2}{2}$ or $\frac{6}{2}::\frac{2}{4}:3\times\frac{2}{4}$ or $\frac{6}{4}$

$\frac{9}{2}$ are $3\times\frac{3}{2}$

$\frac{3}{2}:3\times\frac{3}{2}$ or $\frac{9}{2}::\frac{3}{4}:3\times\frac{3}{4}$ or $\frac{9}{4}$.

And so on.

$\frac{4}{2}$ are 4 times $\frac{1}{2}$

$\frac{1}{2}:4\times\frac{1}{2}$ or $\frac{4}{2}::\frac{1}{4}:4\times\frac{1}{4}$ or $\frac{4}{4}$

$\frac{8}{2}$ are 4 times $\frac{2}{2}$

$\frac{2}{2}:4\times\frac{2}{2}$ or $\frac{8}{2}::\frac{2}{4}:4\times\frac{2}{4}$ or $\frac{8}{4}$

$\frac{12}{2}$ are 4 times $\frac{3}{2}$

$\frac{3}{2}:4\times\frac{3}{2}$ or $\frac{12}{2}::\frac{3}{4}:4\times\frac{3}{4}$ or $\frac{12}{4}$.

And so on.

And so on, as before.

Other combinations of lines may be treated in the same manner.

Questions on Case I.

Second Line, or Line of Halves.

i. $\frac{3}{2}:\frac{9}{2}::\frac{4}{2}:$what number?....*Ans.* $\frac{12}{2}=6$.

Proof. $\frac{9}{2}$ are $3\times\frac{3}{2}$; therefore $\frac{3}{2}:3\times\frac{3}{2}$ or $\frac{9}{2}::\frac{4}{2}:3\times\frac{4}{2}$ or $\frac{12}{2}$.

ii. $\frac{5}{2}:\frac{20}{2}::\frac{3}{2}:$ what number?....*Ans.* $\frac{12}{2}=6$.

Proof. $\frac{20}{2}$ are $4\times\frac{5}{2}$; therefore $\frac{5}{2}:4\times\frac{5}{2}$ or $\frac{20}{2}::\frac{3}{2}:4\times\frac{3}{2}$ or $\frac{12}{2}$.

Second and Third Lines, or Lines of Halves and Thirds.

i. $\frac{3}{2}:\frac{9}{2}::\frac{2}{3}:$ what number?....*Ans.* $\frac{6}{3}=2$.

Proof. $\frac{9}{2}$ are $3\times\frac{3}{2}$; therefore $\frac{3}{2}:3\times\frac{3}{2}$ or $\frac{9}{2}::\frac{2}{3}:3\times\frac{2}{3}$ or $\frac{6}{3}$.

ii. $\frac{1}{2}:2\frac{1}{2}::\frac{4}{3}:$ what number?....*Ans.* $\frac{20}{3}=6\frac{2}{3}$.

Proof. $2\frac{1}{2}=\frac{5}{2}$, $\frac{5}{2}$ are $5\times\frac{1}{2}$; therefore $\frac{1}{2}:5\times\frac{1}{2}$ or $\frac{5}{2}::\frac{4}{3}:5\times\frac{4}{3}$ or $\frac{20}{3}$.

Case II. *When the first term of the proportion is divisible by the second.*

Third Line, or Line of Thirds.

$\frac{1}{3}$ is the $\frac{1}{2}$ of $\frac{2}{3}$

$\frac{2}{3}:\frac{1}{2}$ of $\frac{2}{3}$ or $\frac{1}{3}::\frac{4}{3}:\frac{1}{2}$ of $\frac{4}{3}$ or $\frac{2}{3}$

$\frac{2}{3}$ are $\frac{1}{2}$ of $\frac{4}{3}$

$\frac{4}{3}:\frac{1}{2}$ of $\frac{4}{3}$ or $\frac{2}{3}::\frac{6}{3}:\frac{1}{2}$ of $\frac{6}{3}$ or $\frac{3}{3}$

$\frac{3}{3}$ are $\frac{1}{2}$ of $\frac{6}{3}$

$\frac{6}{3}:\frac{1}{2}$ of $\frac{6}{3}$ or $\frac{3}{3}::\frac{8}{3}:\frac{1}{2}$ of $\frac{8}{3}$ or $\frac{4}{3}$.

And so on.

$\frac{1}{3}$ is the $\frac{1}{3}$ of $\frac{3}{3}$

$\frac{3}{3}:\frac{1}{3}$ of $\frac{3}{3}$ or $\frac{1}{3}::\frac{6}{3}:\frac{1}{3}$ of $\frac{6}{3}$ or $\frac{2}{3}$

$\frac{2}{3}$ are the $\frac{1}{3}$ of $\frac{6}{3}$

$\frac{6}{3}:\frac{1}{3}$ of $\frac{6}{3}$ or $\frac{2}{3}::\frac{9}{3}:\frac{1}{3}$ of $\frac{9}{3}$ or $\frac{3}{3}$.

And so on.

And so on, as before.

Fifth Line, or Line of Fifths.

$\frac{1}{5}$ is the $\frac{1}{2}$ of $\frac{2}{5}$

$\frac{2}{5} : \frac{1}{2}$ of $\frac{2}{5}$ or $\frac{1}{5} :: \frac{6}{5} : \frac{1}{2}$ of $\frac{6}{5}$ or $\frac{3}{5}$

$\frac{2}{5}$ are the $\frac{1}{2}$ of $\frac{4}{5}$

$\frac{4}{5} : \frac{1}{2}$ of $\frac{4}{5}$ or $\frac{2}{5} :: \frac{10}{5} : \frac{1}{2}$ of $\frac{10}{5}$ or $\frac{5}{5}$

$\frac{3}{5}$ are the $\frac{1}{2}$ of $\frac{6}{5}$

$\frac{6}{5} : \frac{1}{2}$ of $\frac{6}{5}$ or $\frac{3}{5} :: \frac{14}{5} : \frac{1}{2}$ of $\frac{14}{5}$ or $\frac{7}{5}$.

And so on.

$\frac{1}{5}$ is the $\frac{1}{3}$ of $\frac{3}{5}$

$\frac{3}{5} : \frac{1}{3}$ of $\frac{3}{5}$ or $\frac{1}{5} :: \frac{12}{5} : \frac{1}{3}$ of $\frac{12}{5}$ or $\frac{4}{5}$

$\frac{2}{5}$ are the $\frac{1}{3}$ of $\frac{6}{5}$

$\frac{6}{5} : \frac{1}{3}$ of $\frac{6}{5}$ or $\frac{2}{5} :: \frac{21}{5} : \frac{1}{3}$ of $\frac{21}{5}$ or $\frac{7}{5}$

$\frac{3}{5}$ are the $\frac{1}{3}$ of $\frac{9}{5}$

$\frac{9}{5} : \frac{1}{3}$ of $\frac{9}{5}$ or $\frac{3}{5} :: \frac{30}{5} : \frac{1}{3}$ of $\frac{30}{5}$ or $\frac{10}{5}$.

And so on.

And so on, as before.

Similar tables may be formed for the proportion of halves, fourths, sixths, &c.

Second and Fourth Lines, or Lines of Halves and Fourths.

$\frac{1}{2}$ is the $\frac{1}{2}$ of $\frac{2}{2}$

$\frac{2}{2} : \frac{1}{2}$ of $\frac{2}{2}$ or $\frac{1}{2} :: \frac{2}{4} : \frac{1}{2}$ of $\frac{2}{4}$ or $\frac{1}{4}$

$\frac{2}{2}$ are the $\frac{1}{2}$ of $\frac{4}{2}$

$\frac{4}{2} : \frac{1}{2}$ of $\frac{4}{2}$ or $\frac{2}{2} :: \frac{4}{4} : \frac{1}{2}$ of $\frac{4}{4}$ or $\frac{2}{4}$

$\frac{3}{2}$ are the $\frac{1}{2}$ of $\frac{6}{2}$

$\frac{6}{2} : \frac{1}{2}$ of $\frac{6}{2}$ or $\frac{3}{2} :: \frac{6}{4} : \frac{1}{2}$ of $\frac{6}{4}$ or $\frac{3}{4}$.

And so on.

$\frac{1}{2}$ is the $\frac{1}{3}$ of $\frac{3}{2}$

$\frac{3}{2} : \frac{1}{3}$ of $\frac{3}{2}$ or $\frac{1}{2} :: \frac{3}{4} : \frac{1}{3}$ of $\frac{3}{4}$ or $\frac{1}{4}$

$\frac{2}{2}$ are the $\frac{1}{3}$ of $\frac{6}{2}$

$\frac{6}{2} : \frac{1}{3}$ of $\frac{6}{2}$ or $\frac{2}{2} :: \frac{6}{4} : \frac{1}{3}$ of $\frac{6}{4}$ or $\frac{2}{4}$

$\frac{3}{2}$ are the $\frac{1}{3}$ of $\frac{9}{2}$

$\frac{9}{2} : \frac{1}{3}$ of $\frac{9}{2}$ or $\frac{3}{2} :: \frac{9}{4} : \frac{1}{3}$ of $\frac{9}{4}$ or $\frac{3}{4}$.

And so on.

$\frac{1}{2}$ is the fourth of $\frac{4}{2}$
$\frac{4}{2} : \frac{1}{4}$ of $\frac{4}{2}$ or $\frac{1}{2} :: \frac{4}{4} : \frac{1}{4}$ of $\frac{4}{4}$ or $\frac{1}{4}$
$\frac{2}{2}$ is the $\frac{1}{4}$ of $\frac{8}{2}$
$\frac{8}{2} : \frac{1}{4}$ of $\frac{8}{2}$ or $\frac{2}{2} :: \frac{8}{4} : \frac{1}{4}$ of $\frac{8}{4}$ or $\frac{2}{4}$
$\frac{3}{2}$ are the $\frac{1}{4}$ of $\frac{12}{2}$
$\frac{12}{2} : \frac{1}{4}$ of $\frac{12}{2}$ or $\frac{3}{2} :: \frac{12}{4} : \frac{1}{4}$ of $\frac{12}{4}$ or $\frac{3}{4}$.
And so on.

And so on, as before.

Other combinations of lines may be treated in the same manner.

Questions on Case II.

Second Line, or Line of Halves.

i. $\frac{14}{2} : \frac{7}{2} :: \frac{8}{2}$: what number ?....*Ans.* $\frac{4}{2} = 2$.

Proof. $\frac{7}{2}$ are $\frac{1}{2}$ of $\frac{14}{2}$; therefore $\frac{14}{2} : \frac{1}{2}$ of $\frac{14}{2}$ or $\frac{7}{2} :: \frac{8}{2} : \frac{1}{2}$ of $\frac{8}{2}$ or $\frac{4}{2}$.

ii. $4\frac{1}{2} : \frac{3}{2} :: \frac{15}{2}$: what number ?....*Ans.* $\frac{5}{2} = 2\frac{1}{2}$.

Proof. $4\frac{1}{2}$ are $\frac{9}{2}$, $\frac{3}{2}$ are $\frac{1}{3}$ of $\frac{9}{2}$; therefore $\frac{9}{2} : \frac{1}{3}$ of $\frac{9}{2}$ or $\frac{3}{2} :: \frac{15}{2} : \frac{1}{3}$ of $\frac{15}{2}$ or $\frac{5}{2}$.

Third Line, or Line of Thirds.

i. $\frac{12}{3} : \frac{3}{3} :: \frac{8}{3}$: what number ?....*Ans.* $\frac{2}{3}$.

Proof. $\frac{3}{3}$ are $\frac{1}{4}$ of $\frac{12}{3}$; therefore $\frac{12}{3} : \frac{1}{4}$ of $\frac{12}{3} :: \frac{8}{3} : \frac{1}{4}$ of $\frac{8}{3}$ or $\frac{2}{3}$.

ii. $\frac{10}{3} : \frac{2}{3} :: 6\frac{2}{3}$: what number ?....*Ans.* $\frac{4}{3} = 1\frac{1}{3}$.

Proof. $6\frac{2}{3}$ are $\frac{20}{3}$, $\frac{2}{3}$ are $\frac{1}{5}$ of $\frac{10}{3}$; therefore $\frac{10}{3} : \frac{1}{5}$ of $\frac{10}{3}$ or $\frac{2}{3} :: \frac{20}{3} : \frac{1}{5}$ of $\frac{20}{3}$ or $\frac{4}{3}$.

Fourth Line, or Line of Fourths.

i. $\frac{6}{4} : \frac{3}{4} :: \frac{14}{4}$: what number ?....*Ans.* $\frac{7}{4} = 1\frac{3}{4}$.

Proof. $\frac{3}{4}$ are $\frac{1}{2}$ of $\frac{6}{4}$; therefore $\frac{6}{4} : \frac{1}{2}$ of $\frac{6}{4}$ or $\frac{3}{4} :: \frac{14}{4} : \frac{1}{2}$ of $\frac{14}{4}$ or $\frac{7}{4}$.

ii. $2\frac{1}{4} : \frac{3}{4} :: 3\frac{3}{4}$: what number ?....*Ans.* $\frac{5}{4} = 1\frac{1}{4}$.

Proof. $2\frac{1}{4}$ are $\frac{9}{4}$, $\frac{3}{4}$ are $\frac{1}{3}$ of $\frac{9}{4}$; therefore $\frac{9}{4} : \frac{1}{3}$ of $\frac{9}{4}$ or $\frac{3}{4} :: \frac{15}{4} : \frac{1}{3}$ of $\frac{15}{4}$ or $\frac{5}{4}$.

Second and Third Lines, or Lines of Halves and Thirds.

i. $\frac{3}{2} : \frac{1}{2} :: \frac{15}{3}$: what number ?.....*Ans.* $\frac{5}{3} = 1\frac{2}{3}$.

Proof. $\frac{1}{2}$ is $\frac{1}{3}$ of $\frac{3}{2}$; therefore $\frac{3}{2} : \frac{1}{3}$ of $\frac{3}{2}$ or $\frac{1}{2} :: \frac{15}{3} : \frac{1}{3}$ of $\frac{15}{3}$ or $\frac{5}{3}$.

ii. $4\frac{1}{2} : 1\frac{1}{2} :: 7\frac{1}{2}$: what number ?.....*Ans.* $\frac{5}{2} = 2\frac{1}{2}$.

Whole Numbers and Fifths.

i. $9 : 3 :: \frac{9}{5}$: what number ?.....*Ans.* $\frac{3}{5}$.

Proof. 3 are $\frac{1}{3}$ of 9; therefore $9 : \frac{1}{3}$ of $9 :: \frac{9}{5} : \frac{1}{3}$ of $\frac{9}{5}$ or $\frac{3}{5}$.

ii. $16 : 4 :: \frac{8}{3}$: what number ?.....*Ans.* $\frac{2}{3}$.

Proof. 4 are $\frac{1}{4}$ of 16; therefore $16 : \frac{1}{4}$ of 16 or $4 :: \frac{8}{3} : \frac{1}{4}$ of $\frac{8}{3}$ or $\frac{2}{3}$.

Case III. *When the first and second terms have any ratio.*

Third Line, or Line of Thirds.

Where the ratio is $5 \times \frac{1}{2}$.

$\frac{5}{3}$ are $5 \times \frac{1}{2}$ of $\frac{2}{3}$

$\frac{2}{3} : 5 \times \frac{1}{2}$ of $\frac{2}{3}$ or $\frac{5}{3} :: \frac{4}{3} : 5 \times \frac{1}{2}$ of $\frac{4}{3}$ or $\frac{10}{3}$

$\frac{10}{3}$ are $5 \times \frac{1}{2}$ of $\frac{4}{3}$

$\frac{4}{3} : 5 \times \frac{1}{2}$ of $\frac{4}{3}$ or $\frac{10}{3} :: \frac{6}{3} : 5 \times \frac{1}{2}$ of $\frac{6}{3}$ or $\frac{15}{3}$

$\frac{15}{3}$ are $5 \times \frac{1}{2}$ of $\frac{6}{3}$

$\frac{6}{3} : 5 \times \frac{1}{2}$ of $\frac{6}{3}$ or $\frac{15}{3} :: \frac{8}{3} : 5 \times \frac{1}{2}$ of $\frac{8}{3}$ or $\frac{20}{3}$.

And so on.

And so on to any other ratio.

Third and Fifth Lines, or Lines of Thirds and Fifths.

$\frac{5}{3}$ are $5 \times \frac{1}{2}$ of $\frac{2}{3}$

$\frac{2}{3} : 5 \times \frac{1}{2}$ of $\frac{2}{3}$ or $\frac{5}{3} :: \frac{2}{5} : 5 \times \frac{1}{2}$ of $\frac{2}{5}$ or $\frac{5}{5}$

$\frac{10}{3}$ are $5 \times \frac{1}{2}$ of $\frac{4}{3}$

$\frac{4}{3} : 5 \times \frac{1}{2}$ of $\frac{4}{3}$ or $\frac{10}{3} :: \frac{4}{5} : 5 \times \frac{1}{2}$ of $\frac{4}{5}$ or $\frac{10}{5}$

$\frac{15}{3}$ are $5 \times \frac{1}{2}$ of $\frac{6}{3}$

$\frac{6}{3} : 5 \times \frac{1}{2}$ of $\frac{6}{3}$ or $\frac{15}{3} :: \frac{6}{5} : 5 \times \frac{1}{2}$ of $\frac{6}{5}$ or $\frac{15}{5}$.

And so on.

And so on to any other ratio.

The preceding tables are constructed to find the fourth term of a proportion when three terms are given; but if two fractions be given to find their proportion, the following form may be used with advantage.

$\frac{3}{7}$ are $3 \times \frac{1}{2}$ of $\frac{2}{7}$

3 are $3 \times \frac{1}{2}$ of 2

$\frac{2}{7} : \frac{3}{7} :: 2 : 3$

$\frac{6}{7}$ are $3 \times \frac{1}{2}$ of $\frac{4}{7}$

3 are $3 \times \frac{1}{2}$ of 2

$\frac{4}{7} : \frac{6}{7} :: 2 : 3$

$\frac{9}{7}$ are $3 \times \frac{1}{2}$ of $\frac{6}{7}$

3 are $3 \times \frac{1}{2}$ of 2

$\frac{6}{7} : \frac{9}{7} :: 2 : 3$.

And so on.

The form of these tables may be varied, to suit particular questions. For instance, $2 : 2\frac{1}{2} :: \frac{8}{3}$: what number?....In this case $2\frac{1}{2}$ are once 2 and the $\frac{1}{4}$ of 2, and 2 : once 2 and $\frac{1}{4}$ of $2 :: \frac{8}{3}$: once $\frac{8}{3}$ and $\frac{1}{4}$ of $\frac{8}{3}$, that is, $\frac{10}{3}$. Again, $\frac{1}{2} : \frac{1}{4} :: \frac{1}{5}$ is to what number?....Here $\frac{1}{4}$ is the $\frac{1}{2}$ of $\frac{1}{2}$; and then,

$\frac{1}{2} : \frac{1}{2}$ of $\frac{1}{2}$ or $\frac{1}{4} :: \frac{1}{5} : \frac{1}{2}$ of $\frac{1}{5}$ or $\frac{1}{10}$.

Questions on Case III.

On the Board of Simple Fractions.

Third Line, or Line of Thirds.

i. $\frac{2}{3} : 1\frac{2}{3} :: 2\frac{2}{3}$: what number?....*Ans.* $\frac{20}{3} = 6\frac{2}{3}$.

Proof. $1\frac{2}{3} = \frac{5}{3}$, $\frac{5}{3}$ are $5 \times \frac{1}{2}$ of $\frac{2}{3}$; therefore $\frac{2}{3} : 5 \times \frac{1}{2}$ of $\frac{2}{3}$ or $\frac{5}{3} :: \frac{8}{3} : 5 \times \frac{1}{2}$ of $\frac{8}{3}$ or $\frac{20}{3}$.

ii. $3\frac{1}{3} : 2\frac{1}{3} :: \frac{30}{3}$: what number?....*Ans.* $\frac{21}{3} = 7$.

iii. $\frac{8}{3} : \frac{20}{3} :: 7$: what number?....*Ans.* $\frac{35}{2} = 17\frac{1}{2}$.

Seventh Line, or Line of Sevenths.

i. What proportion does $\frac{4}{7}$ bear to $\frac{6}{7}$?....*Ans.* 2 to 3.

Proof. $\frac{6}{7}$ are $3 \times \frac{1}{2}$ of $\frac{4}{7}$, 3 are $3 \times \frac{1}{2}$ of 2; therefore $\frac{4}{7} : \frac{6}{7} :: 2 : 3$.

ii. What proportion does $\frac{5}{7}$ bear to $2\frac{1}{7}$?....*Ans.* 1 to 3.

iii. What proportion does $\frac{9}{7}$ bear to $\frac{12}{7}$?....*Ans.* 3 to 4.

iv. What proportion does $\frac{8}{7}$ bear to $2\frac{6}{7}$?....*Ans.* 2 to 5.

v. What proportion does 1 bear to $\frac{9}{7}$?....*Ans.* 7 to 9.

On the Board of Compound Fractions.

Second Line, Third Square.

i. $\frac{2}{3} : \frac{1}{2} :: \frac{4}{3}$: what number?....*Ans.* $\frac{3}{3} = 1$.

Proof. $\frac{2}{3}$ are $\frac{4}{6}$, $\frac{1}{2}$ is $\frac{3}{6}$, $\frac{3}{6}$ are $3 \times \frac{1}{4}$ of $\frac{4}{6}$; therefore $\frac{4}{6} : 3 \times \frac{1}{4}$ of $\frac{4}{6}$ or $\frac{3}{6} :: \frac{4}{3} : 3 \times \frac{1}{4}$ of $\frac{4}{3}$ or $\frac{3}{3}$.

ii. $1 : \frac{1}{2} :: \frac{1}{3}$: what number?....*Ans.* $\frac{1}{6}$.

iii. $\frac{1}{2} : \frac{1}{6} :: \frac{5}{2}$: what number?....*Ans.* $\frac{5}{6}$.

iv. $\frac{1}{3} : \frac{5}{6} :: 1$: what number?....*Ans.* $\frac{5}{2} = 2\frac{1}{2}$.

Miscellaneous Questions.

i. $\frac{3}{8}$ is to $\frac{6}{8}$ as $\frac{2}{5}$ is to what number?....*Ans.* $\frac{4}{5}$.

Proof. $\frac{6}{8}$ are twice $\frac{3}{8}$; therefore $\frac{3}{8}$: twice $\frac{3}{8}$ or $\frac{6}{8}$: : $\frac{2}{5}$: twice $\frac{2}{5}$ or $\frac{4}{5}$.

ii. $\frac{12}{4} : \frac{3}{4} :: \frac{16}{7}$: an unknown number?....*Ans.* $\frac{4}{7}$.

Proof. $\frac{3}{4}$ are $\frac{1}{4}$ of $\frac{12}{4}$; therefore $\frac{12}{4}$: $\frac{1}{4}$ of $\frac{12}{4}$ or $\frac{3}{4}$ :: $\frac{16}{7}$: $\frac{1}{4}$ of $\frac{16}{7}$ or $\frac{4}{7}$.

iii. $\frac{2}{9} : \frac{2}{3} :: \frac{3}{7}$: what number?....*Ans.* $\frac{9}{7}$ or $1\frac{2}{7}$.

Proof. $\frac{2}{3}$ are $\frac{6}{9}$ or 3 times $\frac{2}{9}$. Hence $\frac{2}{9} : 3 \times \frac{2}{9} :: \frac{3}{7} : 3 \times \frac{3}{7}$ or $\frac{9}{7}$.

iv. $\frac{4}{3} : \frac{1}{3} :: \frac{16}{9}$: what number?....*Ans.* $\frac{4}{9}$.

v. $\frac{5}{12} : \frac{10}{4} :: \frac{3}{8}$: an unknown number?....*Ans.* $\frac{18}{8}$ or $2\frac{1}{4}$.

vi. $\frac{6}{7} : \frac{2}{7} :: \frac{18}{20}$: an unknown number?....*Ans.* $\frac{6}{20} = \frac{3}{10}$.

vii. What number has the same proportion to $\frac{2}{7}$ that $\frac{1}{5}$ has to $\frac{21}{15}$?....*Ans.* 2.

viii. $\frac{5}{7} : \frac{5}{28} :: \frac{20}{9}$: what number?....*Ans.* $\frac{5}{9}$.

ix. $1\frac{1}{8} : \frac{3}{8} :: 1\frac{2}{3}$: what number?....*Ans.* $\frac{5}{9}$.

x. $\frac{2}{3} : \frac{1}{4} :: \frac{16}{27}$: what number?....*Ans.* $\frac{2}{9}$.

xi. $\frac{4}{5} : \frac{3}{4} :: \frac{16}{19}$: an unknown number?....*Ans.* $\frac{15}{19}$.

Proof. $\frac{4}{5}$ are $\frac{16}{20}$, $\frac{3}{4}$ are $\frac{15}{20}$, $\frac{15}{20}$ are $15 \times \frac{1}{16}$ of $\frac{16}{20}$; therefore $\frac{16}{20} : 15 \times \frac{1}{16}$ of $\frac{16}{20}$ or $\frac{15}{20} :: \frac{16}{19} : 15 \times \frac{1}{16}$ of $\frac{16}{19}$ or $\frac{15}{19}$.

xii. $\frac{1}{2} : \frac{3}{8} :: \frac{3}{5}$: what number?....*Ans.* $\frac{9}{20}$.

Proof. $\frac{1}{2}$ is $\frac{4}{8}$, $\frac{3}{8}$ are $3 \times \frac{1}{4}$ of $\frac{4}{8}$; $\frac{4}{8} : 3 \times \frac{1}{4}$ of $\frac{4}{8}$ or $\frac{3}{8} :: \frac{3}{5} : 3 \times \frac{1}{4}$ of $\frac{3}{5}$ or $\frac{9}{20}$.

xiii. $\frac{3}{2} : \frac{9}{4} :: 8$: what number?....*Ans.* 12.

xiv. $\frac{1}{5} : \frac{1}{7} :: \frac{1}{9}$: what number?....*Ans.* $\frac{5}{63}$.

xv. $\frac{1}{3} : \frac{2}{5} :: \frac{3}{7}$: what number?....*Ans.* $\frac{18}{35}$.

xvi. $1\frac{3}{4} : 2\frac{1}{3} :: 2\frac{1}{8}$: what number?....*Ans.* $\frac{17}{6}$ or $2\frac{5}{6}$.

xvii. $7\frac{1}{2} : 1 :: 9\frac{1}{4}$: what number?....*Ans.* $\frac{37}{30}$ or $1\frac{7}{30}$.

xviii. $8 : 4 :: \frac{3}{4}$: what number?....*Ans.* $\frac{3}{8}$.

Proof. 4 are $\frac{1}{2}$ of 8; therefore $8 : \frac{1}{2}$ of 8 or $4 :: \frac{3}{4} : \frac{1}{2}$ of $\frac{3}{4}$ or $\frac{3}{8}$.

xix. $4 : 6 :: \frac{2}{3}$: what number ?....*Ans.* 1.

Proof. 6 are $3 \times \frac{1}{2}$ of 4; therefore $4 : 3 \times \frac{1}{2}$ of 4 or $6 :: \frac{2}{3} : 3 \times \frac{1}{2}$ of $\frac{2}{3}$ or 1.

xx. $\frac{1}{2}$ of $\frac{1}{2}$: an unknown number $:: 2 : 8$.....*Ans.* 1.

xxi. What proportion does $3\frac{1}{2}$ bear to $\frac{1}{3}$?....*Ans.* 21 to 2.

Questions on Money, Weights, and Measures.

i. If 15 yards cost 7*s*. 6*d*., what is the amount of $1\frac{1}{2}$ yards ? ....*Ans.* 9*d*.

Proof. $15 = \frac{30}{2}$, and $1\frac{1}{2} = \frac{3}{2}$, $\frac{3}{2}$ are $\frac{1}{10}$ of $\frac{30}{2}$, 7*s*. 6*d*. are 90 pence; hence $\frac{30}{2} : \frac{1}{10}$ of $\frac{30}{2}$ :: 90*d*. : $\frac{1}{10}$ of 90*d*. or 9*d*.

ii. If 20 yards cost 15*s*., what is the price of $1\frac{2}{3}$ yards ? ....*Ans.* 1*s*. 3*d*.

iii. If $3\frac{3}{4}$ yards cost 3*s*. 9*d*., what is the price of $\frac{5}{8}$ of a yard ?....*Ans.* $7\frac{1}{2}$*d*.

iv. If $2\frac{1}{2}$ yards cost 1*l*. 10*s*., what is the cost of $\frac{3}{4}$ of a yard ?....*Ans.* 9*s*.

v. If goods be bought for $7\frac{1}{2}$*d*. and sold for 9*d*., what is the gain upon 60*l*. worth of the same goods ?....*Ans.* 12*l*.

Proof. $7\frac{1}{2}$*d*. taken from 9*d*. leaves $1\frac{1}{2}$*d*. for the gain on $7\frac{1}{2}$*d*.; hence $7\frac{1}{2} : 1\frac{1}{2}$ :: 60*l*. : the gain required, but $7\frac{1}{2} = \frac{15}{2}$, and $1\frac{1}{2} = \frac{3}{2}$, $\frac{3}{2}$ are the $\frac{1}{5}$ of $\frac{15}{2}$; hence $\frac{15}{2} : \frac{1}{5}$ of $\frac{15}{2}$ or $\frac{3}{2}$ :: 60*l*. : $\frac{1}{5}$ of 60*l*. or 12*l*.

vi. If on a shilling I gain 3*d*., what do I gain per cent. ? ....*Ans.* 25*l*.

vii. A ton of tallow cost 20*l*., and was sold for 22*l*. 10*s*., how much per cent. was gained ?....*Ans.* $12\frac{1}{2}$ per cent.

viii. If I buy cloves for 6*s*. 3*d*. per lb., and sell them for 6*s*., how much per cent. is lost ?....*Ans.* 4*l*.

ix. If 7 lbs. cost 3*s*. 6*d*., how much will $17\frac{1}{2}$ lbs. cost ?.... *Ans.* 8*s*. 9*d*.

x. If 6 men working for 9 days earn $4l.$, how much would 3 men earn in 12 days at the same rate of wages? ....*Ans.* $2l.\ 13s.\ 4d.$

Proof. 6 men working for 9 days produce 54 days' work, 3 men for 12 days produce 36 days' work, 36 are $2 \times \frac{1}{3}$ of 54; therefore $56 : 2 \times \frac{1}{3}$ of 56 or $36 :: 4l. : 2 \times \frac{1}{3}$ of $4l.$ or $\frac{8}{3}l.$; $\frac{8}{3}l. = 2\frac{2}{3}l. = 2l.\ 13s.\ 4d.$

xi. A cistern can be filled by 3 pipes: by the first in 2 hours, by the second in 3 hours, and by the third in 4 hours; in what time will the cistern be filled, when the 3 pipes are opened at once?....*Ans.* $\frac{12}{13}$ hours.

Proof. As the first pipe would evidently fill $\frac{1}{2}$ the cistern in 1 hour; the second $\frac{1}{3}$ part in 1 hour; and the third $\frac{1}{4}$ part in 1 hour; they must, together, fill $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$ part in 1 hour, that is, $\frac{13}{12}$ in 1 hour: from this it follows that the $\frac{1}{12}$ of the cistern may be filled in $\frac{1}{13}$ of an hour, and therefore $\frac{12}{12}$, or the whole, in 12 times $\frac{1}{13}$ of an hour, or $\frac{12}{13}$ hours.

xii. How much tea at $6s.$ per lb. must be given for 40 pairs of stockings, at $2s.$ per pair?....*Ans.* $13\frac{1}{3}$ lbs.

Proof. 6 are 3 times 2, that is, a lb. of tea costs 3 times the price of a pair of stockings; therefore, the number of lbs. of tea to be given will equal $\frac{1}{3}$ the number of pairs of stockings; that is, $\frac{1}{3}$ of 40 is $\frac{40}{3} = 13\frac{1}{3}$. Or we have by proportion, $6 : \frac{1}{3}$ of 6 or $2 :: 40 : \frac{1}{3}$ of 40 or $\frac{40}{3}$.

xiii. A and B commenced trade with $200l.$, of which A advanced $150l.$ and B the remainder; they gained $40l.$; required each man's share of the profit?.... *Ans.* A's share $= 30l.$, and B's share $= 10l.$

Proof. $150l.$ are $3 \times \frac{1}{4}$ of $200l.$, that is, A's money is $\frac{3}{4}$ of the whole; but each man's share of the profit must be in proportion to his share of the principal; therefore A's share of the profit will be $\frac{3}{4}$ of $40l. = 30l.$, and therefore B's share $= 40l. - 30l. = 10l.$

TENTH EXERCISE.

Square and Solid Measure.

In this exercise, duodecimals or cross multiplication, will be considered to belong to the subject of fractions. In the first part of the exercise the pupil is shown, how a square unit is formed from a lineal one; and the latter part shows how to obtain the product of two quantities, not expressed in the same lineal unit.

In teaching this exercise, a table of square measure will be found useful. This table is supposed to represent a square foot divided into 144 parts representing square inches.

	1	2	3	4	5	6	7	8	9	10	11	12
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
11												
12												

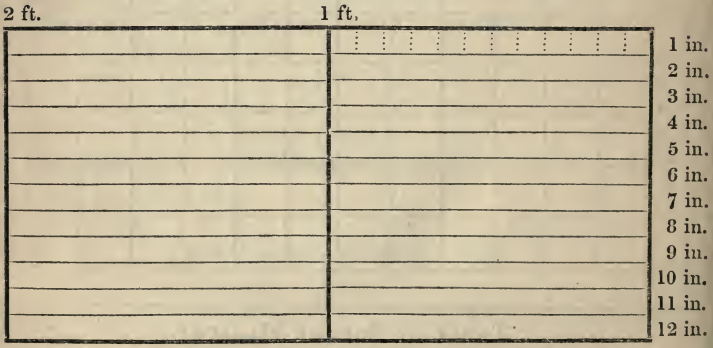
Table of Square Measure.

One square foot contains 144 square inches.

1 in. by 1 in. contains 1 sq. in.	1 in. by 1 in. contains 1 sq. in.
1 in. by 2 in. contains 2 sq. in.	2 in. by 1 in. contain 2 sq. in.
1 in. by 3 in. contains 3 sq. in.	3 in. by 1 in. contain 3 sq. in.
1 in. by 4 in. contains 4 sq. in.	4 in. by 1 in. contain 4 sq. in.
And so on.	And so on.
2 in. by 1 in. contain 2 sq. in.	1 in. by 2 in. contains 2 sq. in.
2 in. by 2 in. contain 4 sq. in.	2 in. by 2 in. contain 4 sq. in.
2 in. by 3 in. contain 6 sq. in.	3 in. by 2 in. contain 6 sq. in.
2 in. by 4 in. contain 8 sq. in.	4 in. by 2 in. contain 8 sq. in.
And so on.	And so on.
3 in. by 1 in. contain 3 sq. in.	1 in. by 3 in. contains 3 sq. in.
3 in. by 2 in. contain 6 sq. in.	2 in. by 3 in. contain 6 sq. in.
3 in. by 3 in. contain 9 sq. in.	3 in. by 3 in. contain 9 sq. in.
3 in. by 4 in. contain 12 sq. in.	4 in. by 3 in. contain 12 sq. in.
And so on.	And so on.
4 in. by 1 in. contain 4 sq. in.	1 in. by 4 in. contains 4 sq. in.
4 in. by 2 in. contain 8 sq. in.	2 in. by 4 in. contain 8 sq. in.
4 in. by 3 in. contain 12 sq. in.	3 in. by 4 in. contain 12 sq. in.
4 in. by 4 in. contain 16 sq. in.	4 in. by 4 in. contain 16 sq. in.
And so on.	And so on.

And so on.

In the same manner it may be shown that feet by feet produce square feet, yards by yards produce square yards, &c.



In the preceding table feet by inches, or inches by feet, are shown to produce twelfths of feet:

12 sq. inches are 1 twelfth of a sq. foot.	1 twelfth of a sq. foot contains 12 sq. inches.

1 ft. by 1 in. contains 1 twelfth of a sq. ft.	1 in. by 1 ft. contains 1 twelfth of a sq. ft.
1 ft. by 2 in. contains 2 ditto	2 in. by 1 ft. contain 2 ditto
1 ft. by 3 in. contains 3 ditto	3 in. by 1 ft. contain 3 ditto
1 ft. by 4 in. contains 4 ditto	4 in. by 1 ft. contain 4 ditto
And so on.	And so on.

2 ft. by 1 in. contain 2 twelfths of a sq. ft.	1 in. by 2 ft. contains 2 twelfths of a sq. ft.
2 ft. by 2 in. contain 4 ditto	2 in. by 2 ft. contain 4 ditto
2 ft. by 3 in. contain 6 ditto	3 in. by 2 ft. contain 6 ditto
2 ft. by 4 in. contain 8 ditto	4 in. by 2 ft. contain 8 ditto
And so on.	And so on.

3 ft. by 1 in. contain 3 twelfths of a sq. ft.	1 in. by 3 ft. contains 3 twelfths of a sq. ft.
3 ft. by 2 in. contain 6 ditto	2 in. by 3 ft. contain 6 ditto
3 ft. by 3 in. contain 9 ditto	3 in. by 3 ft. contain 9 ditto
3 ft. by 4 in. contain 12 ditto	4 in. by 3 ft. contain 12 ditto
And so on.	And so on.

And so on.

Questions on Square Measure.

In proposing the following questions, the teacher is recommended to place before the class the figure, upon which each question is given.

i. How many square feet are contained in 200 square inches ?....*Ans*. 1 square foot and 56 square inches.

ii. How many square inches are contained in a surface which measures 3 inches by 5 inches ?....*Ans*. 15 square inches.

Proof. 3 in. by 1 in. contain 3 square in.; therefore 3 in. by 5 in. will contain 5 times 3 square in., that is, 15 square in.

iii. How many square feet are contained in a surface which measures 4 feet by 3 feet ?....*Ans*. 12 square ft.

iv. What amount of surface is contained by a rectangle, whose length is 2 ft. and breadth 5 inches ?....*Ans*. 10 twelfths of a square ft.

Proof. 1 ft. by 5 in. contains 5 twelfths; and therefore 2 ft. by 5 in. will contain twice 5 twelfths or 10 twelfths.

v. What is the superficial area of a deal board 2 feet by 11 inches ?....*Ans*. 1 square foot and 10 twelfths of a square foot.

Proof. 1 ft. by 11 inches contains 11 twelfths; and therefore 2 ft. by 11 inches, contain twice 11 twelfths or 22 twelfths, 22 twelfths are 1 square ft. and 10 twelfths of a square ft.

vi. The length of a desk is 10 ft., the breadth 2 ft. 3 in., what is its surface ?....*Ans*. 22 square ft. and 6 twelfths.

Proof. 10 ft. by 2 ft. contain 20 square ft., 10 ft. by 3 in. contain 30 twelfths, or 2 square ft. and 6 twelfths; then 20 square ft. + 2 square ft. and 6 twelfths = 22 square ft. and 6 twelfths.

vii. 10 in. by 4 ft. 2 in.....*Ans*. 3 square ft. 5 twelfths and 8 square inches.

Proof. 10 in. by 2 in. contain 20 square in., or 1 twelfth and 8 sq. in.; 10 in. by 4 ft. contain 40 twelfths or 3 square

feet and 4 twelfths; then 3 square ft. and 4 twelfths + 1 twelfth and 8 square in. = 3 square ft., 5 twelfths, and 8 square in.

viii. The length of a door is 6 feet, and the breadth 3 ft. 5 in., what would it cost at the rate of 1*s*. 6*d*. per square ft.? ....*Ans.* 1*l*. 10*s*. 9*d*.

Proof. 6 ft. by 3 ft. 5 in. contain 20 square ft. and 6 twelfths; then 20 square ft. at 1*s*. 6*d*. = 1*l*. 10*s*.; 6 twelfths are $\frac{1}{2}$ of a square foot, $\frac{1}{2}$ of 1*s*. 6*d*. is 9*d*.; therefore the cost would be 1*l*. 10*s*. 9*d*.

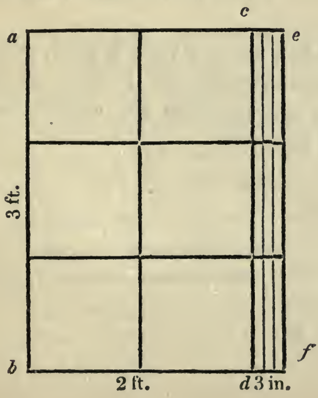
ix. The school yard is 20 feet long by 12 ft. 3 in. broad, what would be the value of the pavement at 1*s*. per square foot?....*Ans.* 12*l*. 5*s*.

x. What would be the cost of flooring a room 21 feet long by 15 feet broad, at 6*s*. per square yard?....*Ans.* 10*l*. 10*s*.

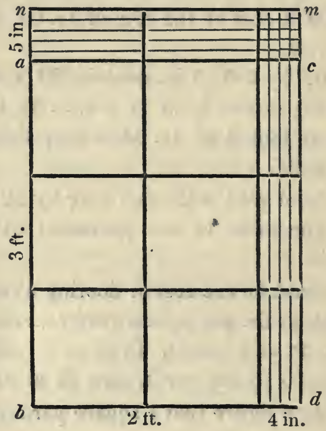
Proof. 21 ft. = 7 yards, 15 ft. = 5 yards, 7 yards by 5 yards, contain 35 square yards, then 35 at 6*s*. = 10*l*. 10*s*.

xi. Prove by a figure that 1 square yard contains 9 square feet.

xii. Represent the product of 2 feet 3 in. by 3 feet.....*Ans.* The space *a b c d* represents the product of 2 feet by 3 feet = 6 square feet: *c d e f* the product of 3 in. by 3 ft. = 9 twelfths, so that the whole rectangle *a b e f*, which is 2 ft. 3 in. by 3 ft. = 6 square ft. and 9 twelfths.



xiii. Give a representation of the product of 2 feet 4 in. by 3 ft. 5 in.....*Ans.* The space *a b c d* represents the product of 2 ft. 4 in. by 3 ft. = 6 sq. ft. and 12 twelfths = 7 sq. ft.;



n a m c the product of 2 ft. 4 in. by 5 in. = 10 twelfths and 20 sq. in. = 11 twelfths and 8 sq. in.; and the whole rectangle *n b m d*, which is 2 ft. 4 in. by 3 ft. 5 in. = 7 sq. ft. 11 twelfths and 8 sq. in.

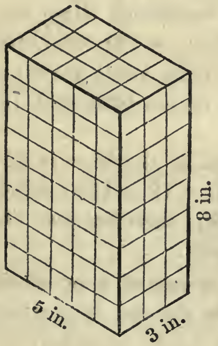
xiv. What is the area of a board whose length is 3 ft. 2 in. and breadth 2 ft. 5 in.?....*Ans.* 7 sq. ft. 7 twelfths and 10 sq. in.

Proof. 3 ft. 2 in. by 2 ft. = 6 sq. ft. and 4 twelfths; 3 ft. 2 in. by 5 in. = 15 twelfths and 10 sq. in. = 1 sq. ft. 3 twelfths and 10 sq. in.; and therefore the whole area will be 7 sq. ft. 7 twelfths and 10 sq. in.

Questions on Solid Measure.

In proposing the following questions, it is desirable that some of the more important solids should be placed before the class.

i. A rectangular block is 5 in. by 3 in. in the base, and 8 in. long, how many inch cubes could be cut out of it?.... *Ans.* 120.



Proof. The base contains 15 sq. in.; and if the block were an inch in height it would contain exactly 15 inch cubes, so that in every inch of length there would be 15 inch cubes; therefore for 8 inches in length we have 8 × 15, or 120 inch cubes.

ii. How many cubic inches are contained in a cubic foot? ....*Ans.* 1728 cubic inches make one cubic foot.

Proof. The base of the cube being 12 in. by 12 in., contains 144 sq. in., so that each section of an inch in height contains 144 inch cubes; and therefore 12 inches in height contains 12 times 144, or 1728 inch cubes.

iii. How many foot cubes are contained in a cube whose side is a yard?....*Ans.* 27 cubic feet make one cubic yard.

Proof. The base of the cube being 3 ft. by 3 ft., contains 9 sq. ft.; if the solid were a foot high, it would contain 9 foot cubes; and therefore when the solid is 3 ft. high, it contains 3 times 9, or 27 foot cubes.

iv. The base of a block of stone is 2 ft. by 3 ft., and the length 5 ft.; how many cubic feet does it contain ?....*Ans.* 30 cubic ft.

Proof. The area of the base contains 6 sq. ft., which, multiplied by the length, gives 30 cubic ft. for the contents of the solid.

v. The base is 2 in. by 5 in., and the height 4 feet; required the solid contents.....*Ans.* 480 cubic inches.

Proof. The base contains 10 sq. in.; 4 feet = 48 inches; then 10 sq. in. × 48 in. = 480 cubic inches.

vi. How many cubic yards are contained in a cutting of earth 10 ft. by 9 ft. in the base, and 12 ft. high ?....*Ans.* 40 cubic yards.

Proof. The area of the base is 90 sq. ft.; and the solid contents = 90 sq. ft. × 12 = 1080 cubic ft.; but 1 cubic yd. contains 27 cubic ft.; therefore the twenty-seventh part of 1080 = 40 cubic yards.

vii. What is the cost of a log of timber 20 ft. long, and 1 ft. 6 in. by 1 ft. in the end or section, at 1*s.* 6*d.* per cubic foot ?....*Ans.* 2*l.* 5*s.*

Proof. The area of the end = 1 ft. 6 in. by 1 ft. = 1 sq. ft. and 6 twelfths = $1\frac{1}{2}$ sq. ft.; and the solid contents = $1\frac{1}{2} \times 20 = 30$ cubic ft.; then 30 at 1*s.* 6*d.* = 2*l.* 5*s.*

viii. What is the cost of a block of marble 6 feet long, 2 ft. broad, and 6 in. thick, at 6*s.* 6*d.* per cubic foot ?.... *Ans.* 1*l.* 19*s.*

ix. What is the weight of a rectangular block of stone, 10 feet long, 4 feet broad, and 2 feet thick, when 1 cubic foot weighs 130 lbs. ?....*Ans.* 10,400 lbs.

Proof. The solid contents = 4 × 2 × 10 = 80 cubic feet; then 80 times 130 lbs. = 10,400 lbs.

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